NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2416

THEORETICAL STUDY OF SOME METHODS FOR INCREASING THE SMOOTHNESS OF FLIGHT THROUGH ROUGH AIR

By William H. Phillips and Christopher C. Kraft, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
July 1951

TABLE OF CONTENTS

Page

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2416

THEORETICAL STUDY OF SOME METHODS FOR INCREASING THE SMOOTHNESS OF FLIGHT THROUGH ROUGH AIR

By William H. Phillips and Christopher C. Kraft, Jr.

SUMMARY

A theoretical study, based on the usual assumptions of airplane stability theory, has been made of the response to gusts and the stability and control characteristics of an airplane equipped with systems in which wing flaps and elevators are operated to reduce accelerations in rough air. These surfaces are assumed to be actuated by an automatic control system in response to the indications of an angle-of-attack vane or an accelerometer. The effect of interconnection of the flap-operating mechanism with the pilot's control as a means of overcoming the adverse effects of these systems on the control characteristics of the airplane is investigated. Limitations of the theory are discussed. Formulas are given for calculating the response to gusts, the response to control movements, and the static and dynamic stability characteristics of airplanes equipped with these devices.

The analysis shows that flaps with characteristics similar to those of conventional landing flaps are unsuitable for reducing acclerations due to gusts. Use of such flaps produces excessive pitching motion of the airplane in rough air and results in unsatisfactory dynamic stability characteristics. In order to be most effective in reducing airplane motion in rough air, the flaps should produce zero pitching moment about the wing aerodynamic center and downwash at the tail in the direction opposite from that normally expected. Means for providing these characteristics in practice are suggested. Flaps having these properties are shown to be very effective in reducing accelerations in rough air when used in conjunction with either the vane or accelerometer sensing device. The stability and control characteristics of the arrangement that is theoretically optimum for reduction of accelerations due to gusts may be unsatisfactory but, if this arrangement is slightly modified to provide increased static stability, both the control characteristics and dynamic stability characteristics appear desirable. Interconnection of the flap-operating mechanism and the pilot's control results in a more rapid response to control deflection than is obtained on conventional airplanes.

In this theoretical study no attempt has been made to consider engineering problems involved in the design of an actual mechanism to reduce the accelerations of an airplane in rough air.

INTRODUCTION

The reduction of accelerations caused by rough air would be of obvious value for improving passenger comfort in commercial airline operation. An airplane capable of smooth flight through rough air would also be a valuable tool for studying the gust structure of the atmosphere.

Previous studies of devices for reducing the accelerations caused by gusts (usually termed gust alleviators) have been made with the object of reducing the structural loads due to the most abrupt and severe gusts. Inasmuch as the provision of such devices complicates the normal control of the airplane, some proposed devices have been designed to come into effect only when certain limits of gust severity were exceeded. Such devices would be of little value for improving passenger comfort. In other studies of gust alleviators the problems of stability and control have not been seriously considered.

The present paper contains a theoretical analysis of various means for increasing the smoothness of flight through rough air. Emphasis has been placed on reduction of accelerations rather than on reduction of structural stresses. An analysis is presented of systems in which the wing flaps and elevator are operated through an automatic control system in response to the indications of an angle-of-attack vane or an accelerometer. The aerodynamic characteristics of such controls required to provide smooth flight through rough air are derived. The response to gusts and the stability and control characteristics of airplanes equipped with these systems are investigated. The effect of interconnecting the flap-operating mechanisms with the pilot's control as a means of overcoming the adverse effects of these systems on the control characteristics of the airplane is also studied.

Because of the emphasis placed on reduction of accelerations rather than structural loads, the devices considered in this paper have been called "acceleration alleviators." This paper is confined to the development of a theoretical basis for the design of acceleration alleviators. Engineering problems involved in the design of an actual mechanism are not considered.

REQUIREMENTS FOR PROVISION OF SMOOTH FLIGHT

Factors Influencing Passenger Comfort and Safety

The design of a device for providing comfortable flight for the passengers of an airplane in rough air requires a knowledge of the factors which contribute to passenger comfort. Unfortunately, very little quantitative information is available as to the types of motion or other stimuli that are most often responsible for airsickness. A review of the available information on this subject is given by McFarland in reference 1. The information indicated that slow oscillations of large amplitude are more likely to cause sickness than faster oscillations of small amplitude. This statement is based mainly on a series of tests, reported in reference 2, in which a large number of men were subjected to vertical oscillations in a device similar to an elevator. With this device, the wave form, the amplitude, and the period of the oscillation could be varied. Some of the results obtained in these tests are shown in figure 1. This figure shows the percentage of the men tested who became sick within a period of 20 minutes when they were subjected to oscillations of each of the types shown. The results of these tests showed that very little sickness was produced by the shortest-period oscillation tested, which had a period of 1.87 seconds. The incidence of sickness reached a maximum at periods of 3 to 4 seconds and decreased slightly at the longest period tested, 4.62 seconds. In the data shown in figure 1, the magnitude of the velocity at the midpoint of each cycle was kept constant as the period increased, so that the average acceleration over the cycle decreased with increasing period. If the acceleration were kept constant as the period increased, still further adverse effect of the longer-period oscillations would be expected.

The results of the tests shown in figure 1, while they do not cover a very wide range of conditions, appear to be in accord with common experience on the subject. Thus, a periodic motion of a small boat in a rough sea is known to produce sickness in a relatively large percentage of passengers, whereas the motion of a trolley car on a rough track, which involves abrupt jolts and jerks containing components of oscillation of high frequency and fairly large amplitude, causes sickness in very few passengers. Reduction of the more prolonged changes in vertical acceleration would therefore appear to be beneficial for minimizing airsickness.

Other stimuli may be important in the production of motion sickness. These stimuli include lateral or rotational accelerations, motion of objects in the field of vision, and many psychological factors such as noise, vibration, temperature, ventilation, and so forth. The relative importance of lateral and normal accelerations in producing motion sickness has apparently not been established. In view of these factors, it may be expected that even complete elimination of normal accelerations would not necessarily eliminate airsickness. The available evidence indicates, however, that periodic changes in normal acceleration are a major cause of motion sickness. In flight through rough air, furthermore, the changes in normal acceleration are relatively large, whereas lateral accelerations, rotational accelerations, and changes in orientation are relatively small. Reduction of the changes in normal acceleration would therefore appear to be the most promising method of improving passenger comfort.

Reduction of changes in vertical acceleration will also reduce the probability of passengers being thrown from their seats by unexpected severe down gusts. A continuously operating device would not be required to prevent this occurrence, but if a continuously operating device is installed for the improvement of passenger comfort, it will provide this result as an additional benefit. Reduction of the more prolonged changes in vertical acceleration would be of greater importance in this connection also, because the distance a passenger is thrown from his seat would increase with the length of time the acceleration was applied.

Methods for Reducing Airplane Motion in Rough Air

Ideally, the airplane should fly in a straight line with no rotation about any axis. The conventional automatic pilot attempts to prevent rotations in roll, pitch, and yaw. Present-day autopilots generally do not have sufficiently rapid response to suppress completely rotations due to gusts, but, by increasing the speed of response of the servomechanisms, this condition could in principle, at least, be closely approached. Elimination of rotations, however, does not prevent vertical motions of the airplane. In fact, maintaining the airplane at a constant angle of pitch increases the response somewhat to low-frequency gusts because a stable airplane tends to relieve changes in acceleration by pitching into the gusts. In order to avoid the vertical accelerations due to rough air, the additional lift caused by a change in angle of attack from the steady flight condition must be eliminated. The following methods might be considered to accomplish this result:

(a) Pitching the whole airplane to maintain a constant angle of attack during passage through the gusts

(b) Variation of wing incidence to maintain a constant angle of attack during passage through the gusts

(c) Operation of flaps or other controls to offset the lift increments on the wing

Method (a) has the advantage that it may be accomplished by the use of the elevators without provision of additional controls on the aircraft. The theoretical possibilities of this method will be discussed in a subsequent section of the paper.

In connection with methods (b) and (c), a problem of longitudinal control arises. Normally, control of the airplane by the elevators is accomplished by pitching the whole airplane to change the angle of attack. If the lift increment due to change of angle of attack is eliminated, the elevators will be ineffective for producing a change in the direction of the flight path. This problem has not been given detailed consideration in most previous investigations of gust-alleviating devices.

Most schemes considered in the past have employed a partial utilization of method (b). If the primary object is to reduce wing root stresses instead of accelerations, a device located near the wing tip may accomplish this result without offsetting the entire lift increment of the wing. By offsetting only part of the lift increment due to angle of attack, the problem of loss of elevator control is avoided to some extent. Devices operating on this principle by utilizing wing torsion, bending of sweptback wings, or deflection of wings mounted on skewed, spring-loaded hinges are described in references 3, 4, and 5. All these methods are seriously limited in their application, however, because they interfere with the provision of adequate lateral control. With all these methods, deflection of the ailerons results in an antisymmetrical angle-of-attack distribution due to distortion of the wing which produces a rolling moment opposing that from the ailerons. As a result, the lateral-control effectiveness may be reduced and the aileron reversal speed excessively lowered.

The use of flaps or other controls to offset the lift increments on the wing during passage through gusts has been proposed by various investigators. Descriptions or investigations of some of these arrangements are given in references 6 to 10.

A review of the systems mentioned in the previous paragraphs indicates that the most promising method of accomplishing the present purpose of reducing accelerations due to rough air is the operation of trailing-edge flaps by an automatic control system to offset continuously the lift increments due to gusts. In order to avoid interference with the aileron control, such flaps should deflect symmetrically. These flaps would be operated to tend to maintain the center of gravity of the airplane on a straight path. The rolling motions due to asymmetrical gusts could be offset by an independent control system consisting of a suitable automatic pilot connected to the ailerons sensitive to angle of roll and its derivatives. In general, the trailing-edge flaps could be installed most conveniently in the same location as conventional landing flaps, that is, inboard of the ailerons. If an irreversible power-boost aileron control system is employed, however, there is a possibility that the ailerons might be deflected symmetrically to assist in gust alleviation without interfering with the pilot's lateral control forces.

The flaps might be operated by a mechanism sensitive to changes in acceleration or to other quantities related to changes in acceleration such as wing-spar stresses, wing-pressure differentials, or the indications of gust detectors placed ahead of the nose or wings. At first glance, it would appear that a mechanism sensitive to changes in acceleration should be the most effective for reducing these changes because it could counteract accelerations from any type of gust. A detector placed ahead of the nose or wings has the advantage that it gives a small amount of anticipation of the action of the gusts. Utilization of this effect may simplify the design of the system by reducing the required rate of response of the control mechanism.

An experimental investigation (reference 11) has been made to determine whether a single angle-of-attack vane mounted ahead of the nose would give a sufficiently accurate measure of the average angle of attack caused by gusts over the entire wing span. The results indicated that such a detector would be of value as a sensing device for operating an acceleration alleviator, though rapid fluctuations caused by small-scale turbulence would have to be filtered from its output. Both the vane and accelerometer sensing devices will be considered in subsequent sections of this paper.

SYMBOLS

b wing span

c mean aerodynamic chord of wing

C_L lift coefficient $\left(\frac{L}{\frac{1}{2}\rho V^2 S}\right)$

C_m pitching-moment coefficient $\left(\frac{M}{\frac{1}{2}\rho V^2 Sc}\right)$

C_Z vertical-force coefficient $\left(\frac{Z}{\frac{1}{2}\rho V^2 S}\right)$

D differential operator (d/ds)

F	Froude number (gc/V^2)
g	acceleration due to gravity
k_y	radius of gyration of airplane about Y-axis
K_y	nondimensional radius-of-gyration factor (k_y/c)
K	ratio between flap deflection and quantity measured by gust detector with elevator fixed
L	lift
l	ratio of tail length to mean aerodynamic chord of wing
l_n	ratio of distance between angle-of-attack vane and center of gravity to mean aerodynamic chord of wing
m	mass of airplane, or ratio between flap deflection and elevator deflection with quantity measured by gust detector fixed
M	pitching moment (positive upward)
n	number of g normal acceleration
N	frequency, cycles per second
P	period of oscillation
q	pitching velocity $(\dot{\theta})$
A	aspect ratio
s	distance measured in chords (tV/c)
S	wing area
t	time
$T_{1/2}$	time to damp to one-half amplitude
T_2	time to double amplitude
V	velocity of center of gravity of airplane with respect to still air
w	velocity along Z-axis

w_g	vertical velocity of gust (positive upward)
w_{g_a}	half-amplitude of vertical velocity of gust
x	distance along horizontal axis
Z	vertical force (positive downward)
α_g	angle of attack due to gust $\left(w_g/V\right)$
α_{g_a}	half-amplitude of angle of attack due to gust
α_o	angle between X-axis and velocity vector V
α_t	angle of attack of tail
α_w	angle of attack of wing
δ_e	elevator deflection
δ_f	flap deflection
δ_v	deflection of angle-of-attack vane
$\Delta_0, \Delta_1, \Delta_2, \ldots$	determinants
ϵ	downwash angle at tail
θ	angle of pitch
ρ	air density
τ	nondimensional time lag of servomechanism, expressed in chords traveled
μ	relative-density factor $(m/\rho Sc)$
ω	nondimensional circular frequency $(2\pi Nc/V)$
λ	wave length

Subscripts:

w	wing
t	tail

f flap

v vane

e elevator

Dot over quantity indicates differentiation with respect to time.

Stability derivatives indicated by subscript notation; for example, $C_{Z_\alpha} = \frac{dC_Z}{d\alpha}$. Rotary derivatives are defined as indicated by the following examples:

$$C_{Z_q} = \frac{\partial C_Z}{\partial\left(\frac{qc}{2V}\right)} \qquad C_{Z_{D\alpha}} = \frac{\partial C_Z}{\partial\left(\frac{\dot{\alpha}c}{2V}\right)}$$

$$C_{Z_{Dq}} = \frac{\partial C_Z}{\partial\left(\frac{\dot{q}c^2}{4V^2}\right)} \qquad C_{Z_{D^2\alpha}} = \frac{\partial C_Z}{\partial\left(\frac{\ddot{\alpha}c^2}{4V^2}\right)}$$

$$C_{Z_{D\delta_f}} = \frac{\partial C_Z}{\partial\left(\frac{\dot{\delta}_f c}{2V}\right)}$$

Subscript following a stability derivative indicates component of airplane which contributes the derivative; for example,

$\left(C_{m_{\delta_f}}\right)_w$ variation of pitching-moment coefficient with flap deflection contributed by the wing

$\left(C_{Z_{\delta_e}}\right)_t$ variation of vertical-force coefficient with elevator deflection contributed by the tail

THEORETICAL ANALYSIS

Method of Analysis

The effectiveness of various acceleration-alleviation systems is studied by calculating the response of airplanes to steady sinusoidal gust disturbances of various frequencies. The use of sinusoidal disturbances appears to be appropriate for the study of the effect of rough air on passenger comfort, because flight records of the accelerations experienced in rough air show that the disturbances are of an irregular oscillatory nature. An irregular disturbance of this type could, if desired, be resolved into sinusoidal components of various frequencies and amplitudes and the resulting motion of the airplane calculated by superimposing the responses to the individual disturbances.

The equations of motion are set up in a form applicable to the analysis of all the acceleration-alleviation systems considered in this paper. In order to illustrate the use of these equations, expressions for the accelerations and pitching velocities experienced by the basic airplane in flight through sinusoidal gusts of various frequencies are derived in this section. The methods of deriving similar expressions for airplanes with acceleration-alleviation systems are presented in subsequent sections of the paper.

The method of calculation of the response of the airplane to gusts used herein is essentially the same as that developed by Wilson in NACA Rep. 1 and other early reports (reference 12). The theory presented in these reports was based on the classical airplane stability theory of Bryan (reference 13) and Bairstow (reference 14), which has formed the basis for most subsequent airplane stability calculations. These theories neglect any unsteady-lift effects. Most later investigations of the response of airplanes to gusts, such as that of Küssner (reference 15), have been concerned with the structural loads caused by flying into a sharp-edge gust or a gust with a steep gradient. For problems of this type, lag in the build-up of lift following penetration of a sharp-edge gust and flexibility of the airplane structure are important. In the present analysis, which must be kept as simple as possible in order to allow the investigation of a large number of variables, it is very desirable to avoid the complication introduced by including these effects. A discussion of the magnitude of the errors caused by neglect of these factors will be given subsequently.

Equations of Motion

The development of the equations of longitudinal motion is generally similar to that given in previous reports on longitudinal stability (for example, see reference 16). A variation from the usual procedure is made, however, by expressing separately the forces and moments contributed by the wing-fuselage combination, the tail, the flaps, and the elevators. The contributions of each of these items to the stability derivatives of the entire airplane may then be easily determined from the subsequent analysis. While this procedure is not essential in setting up the equations for the basic airplane, it is advantageous in studying the effects of the more complex acceleration-alleviator systems.

The equations are set up with respect to body axes. The definitions of the symbols and axes are given in figure 2. The airplane weight and the steady lift force required to balance it are omitted from the equations at present because only changes from the steady trim condition are considered. Thus, each of the variables is defined as the change in this quantity from its steady value. The airplane is assumed to be in horizontal flight. Changes in forward velocity are neglected because the gust disturbances to be considered have periods so short compared with the period of the phugoid motion of the airplane that no appreciable variation in forward speed can occur. More detailed justification for this assumption in problems involving short-period oscillations is given in reference 17.

The equations take the following form:

$$\left.\begin{aligned} m(\dot{w} - V\dot{\theta}) &= \alpha_w \frac{\partial Z}{\partial \alpha_w} + \alpha_t \frac{\partial Z}{\partial \alpha_t} + \delta_f \frac{\partial Z}{\partial \delta_f} + \delta_e \frac{\partial Z}{\partial \delta_e} \\ mk_y{}^2\ddot{\theta} &= \alpha_w \frac{\partial M}{\partial \alpha_w} + \alpha_t \frac{\partial M}{\partial \alpha_t} + \delta_f \frac{\partial M}{\partial \delta_f} + \delta_e \frac{\partial M}{\partial \delta_e} \end{aligned}\right\} \tag{1}$$

<u>Nondimensionalizing of equations</u>.- To nondimensionalize these equations, the following substitutions are made:

$$s = \frac{tV}{c}$$

$$D = \frac{d}{ds} = \frac{c}{V}\frac{d}{dt}$$

$$K_y = \frac{k_y}{c}$$

$$\mu = \frac{m}{\rho Sc}$$

In addition, the vertical force and moment are expressed in coefficient form in accordance with the usual conventions

$$C_Z = \frac{Z}{\frac{\rho}{2} V^2 S} \qquad C_m = \frac{M}{\frac{\rho}{2} V^2 Sc}$$

The equations then become

$$\left.\begin{aligned}
2\mu\left(D\frac{w}{V} - D\theta\right) &= \alpha_w C_{Z_{\alpha_w}} + \alpha_t C_{Z_{\alpha_t}} + \delta_f\left(C_{Z_{\delta_f}}\right)_w + \delta_e\left(C_{Z_{\delta_e}}\right)_t \\
2\mu K_y{}^2 D^2\theta &= \alpha_w C_{m_{\alpha_w}} + \alpha_t C_{m_{\alpha_t}} + \delta_f\left(C_{m_{\delta_f}}\right)_w + \delta_e\left(C_{m_{\delta_e}}\right)_t
\end{aligned}\right\} \qquad (2)$$

Note that in these equations the coefficients of tail moment and vertical force, $C_{m_{\alpha_t}}$, $C_{Z_{\alpha_t}}$, and so forth, are based on wing area and wing chord.

In order to obtain from these equations the response of an airplane to gusts, the values of α_w and α_t must be expressed in terms of the gust velocity and the airplane motion. Derivation of the expressions for α_w and α_t requires consideration of the assumed form of gust disturbance and the downwash effects at the tail.

Form of gust disturbance.- The gust disturbance is assumed to consist of alternate regions of upward and downward vertical velocity in the path of the airplane. The gust velocities are assumed not to change with time and to be uniform across the span of the airplane. The assumption of vertical gusts was made because it is shown in reference 15 that the vertical component of gust velocity has a predominant effect on the accelerations experienced by an airplane.

The equations are solved for the quantities w/V and $D\theta$ in terms of the gust velocity. The normal acceleration, which is proportional to the quantity $D\frac{w}{V} - D\theta$, may then be calculated. In order to obtain this solution, the quantities α_w and α_t must be expressed in terms of these variables. A gust of vertical velocity w_g will cause a change in angle of attack of the wing equal to w_g/V. In addition, the wing may have an angle of attack relative to still air equal to w/V. Let

$$\left.\begin{aligned}
\alpha_g &= \frac{w_g}{V} \\
\alpha_o &= \frac{w}{V}
\end{aligned}\right\} \qquad (3)$$

then the total angle of attack of the wing is

$$\alpha_W = \alpha_o + \alpha_g \tag{4}$$

Approximation to downwash at tail.- In stability theory, the angle of attack of the tail is ordinarily assumed to equal the angle of attack of the wing minus the downwash angle resulting from the wing lift which existed when the wing was at the position now occupied by the tail. There is therefore a time lag, equal to the time required for the airplane to travel one tail length, between the occurrence of a given angle of attack of the wing and the occurrence of the associated downwash at the tail. The effect of this lag in downwash was first pointed out by Cowley and Glauert (reference 18). When the effect of penetrating a gust is considered, a similar lag may be seen to exist between the effects of the gust on the wing and on the tail. The expression for the angle of attack of the tail, expressed in nondimensional notation, is therefore

$$\alpha_t = \alpha_o - \left(\alpha_o \frac{\partial\epsilon}{\partial\alpha}\right)_{s=-l} + \left(\alpha_g\right)_{s=-l} - \left(\alpha_g \frac{\partial\epsilon}{\partial\alpha}\right)_{s=-l} - \left(\delta_f \frac{\partial\epsilon}{\partial\delta_f}\right)_{s=-l} + l\,D\theta$$

A constant time lag l in the occurrence of a quantity may be expressed mathematically by multiplication by the operator e^{-lD} (reference 19). The angle of attack at the tail may therefore be written

$$\alpha_t = \alpha_o - \alpha_o \frac{\partial\epsilon}{\partial\alpha} e^{-lD} + \alpha_g e^{-lD} - \alpha_g \frac{\partial\epsilon}{\partial\alpha} e^{-lD} - \delta_f \frac{\partial\epsilon}{\partial\delta_f} e^{-lD} + l\,D\theta \tag{5}$$

Use of the operator e^{-lD} leads to a transcendental equation which is rather difficult to handle. For this reason, previous investigations dealing with a time lag l in connection with other problems have used a power series expansion of e^{-lD}, retaining only as many terms as considered necessary (see references 19 and 20). This expansion is

$$e^{-lD} = 1 - l\,D + \frac{l^2D^2}{2!} - \frac{l^3D^3}{3!} \cdot \cdot \cdot$$

In the original analysis by Cowley and Glauert (reference 18) in which the basic idea of a lag in downwash was introduced, the effect of this time lag on the downwash at the tail was calculated only for the case of a constant rate of change of angle of attack. For this case the downwash at the tail in still air may be shown to be

$$\alpha_t = \alpha_o - \alpha_o \frac{\partial \epsilon}{\partial \alpha} (1 - l\ D)$$

This expression is equivalent to the first two terms of the series expansion of e^{-lD}.

Since the publication of the report by Cowley and Glauert, theories based on this expression have been almost universally used in airplane stability calculations. A later analysis by Jones and Fehlner (reference 21), which takes into account the actual development of vorticity in the wake, has shown that the operator e^{-lD}, while it correctly represents the constant-time-lag concept, gives only an approximation to the true downwash variation at the tail during an oscillation. An unpublished analysis by the first author of the present paper has shown that the approximate expression $1 - l\ D$ actually predicts the downwash more accurately at frequencies near the frequency of the short-period oscillation of the airplane than does either the expression e^{-lD} or any of the series expansions of e^{-lD} containing higher-order terms. This fact probably explains the success of Cowley and Glauert's approximation in stability calculations.

At still higher oscillation frequencies, the expression $1 - l\ D$ gives the magnitude of the downwash variation more accurately than does e^{-lD}, but gives a phase angle which is increasingly in error as the frequency is increased.

Because of the desirability of simplifying the present analysis as much as possible, the expression $1 - l\ D$ will be used in place of the operator e^{-lD} in equation (5). The foregoing paragraph indicates that this substitution may improve the accuracy of the terms involving downwash, particularly at low frequencies. The term e^{-lD}, however, represents a true time lag between the penetration of the gust by the wing and the tail. Use of the expression $1 - l\ D$ therefore decreases the accuracy of this term.

The expression for the angle of attack of the tail then becomes

$$\alpha_t = \alpha_o - \alpha_o \frac{\partial \epsilon}{\partial \alpha} (1 - l\ D) + \alpha_g (1 - l\ D) - \alpha_g \frac{\partial \epsilon}{\partial \alpha} (1 - l\ D) - \delta_f \frac{\partial \epsilon}{\partial \delta_f} (1 - l\ D) + l\ D\theta \tag{6}$$

Final form of equations.- If the expressions (3), (4), and (6) are substituted in equations (2) and the coefficients of α_o, $D\alpha_o$, and so forth are collected, the equations may be written in the following form:

$$\left.\begin{aligned}
&2\mu D\left(\alpha_o - \theta\right) - \alpha_o C_{Z_\alpha} - \frac{1}{2}\, D\alpha_o\, C_{Z_{D\alpha}} - \frac{1}{2}\, D\theta\, C_{Z_q} - \delta_f C_{Z_{\delta_f}} - \frac{1}{2}\, D\delta_f\, C_{Z_{D\delta_f}} - \delta_e C_{Z_{\delta_e}} = \\
&\qquad \alpha_g C_{Z_\alpha} + \frac{1}{2}\, D\alpha_g\left(C_{Z_{D\alpha}} - C_{Z_q}\right) \\
&2\mu K_y{}^2 D^2\theta - \alpha_o C_{m_\alpha} - \frac{1}{2}\, D\alpha_o\, C_{m_{D\alpha}} - \frac{1}{2}\, D\theta\, C_{m_q} - \delta_f C_{m_{\delta_f}} - \frac{1}{2}\, D\delta_f\, C_{m_{D\delta_f}} - \delta_e C_{m_{\delta_e}} = \\
&\qquad \alpha_g C_{m_\alpha} + \frac{1}{2}\, D\alpha_g\left(C_{m_{D\alpha}} - C_{m_q}\right)
\end{aligned}\right\}\tag{7}$$

where

$$\left.\begin{aligned}
C_{Z_\alpha} &= C_{Z_{\alpha_w}} + \left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) C_{Z_{\alpha_t}} & C_{m_\alpha} &= C_{m_{\alpha_w}} + \left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) C_{m_{\alpha_t}} \\
C_{Z_{D\alpha}} &= 2l\, \frac{\partial\epsilon}{\partial\alpha}\, C_{Z_{\alpha_t}} & C_{m_{D\alpha}} &= 2l\, \frac{\partial\epsilon}{\partial\alpha}\, C_{m_{\alpha_t}} \\
C_{Z_q} &= 2l C_{Z_{\alpha_t}} & C_{m_q} &= 2l C_{m_{\alpha_t}} \\
C_{Z_{\delta_f}} &= \left(C_{Z_{\delta_f}}\right)_w - \frac{\partial\epsilon}{\partial\delta_f}\, C_{Z_{\alpha_t}} & C_{m_{\delta_f}} &= \left(C_{m_{\delta_f}}\right)_w - \frac{\partial\epsilon}{\partial\delta_f}\, C_{m_{\alpha_t}} \\
C_{Z_{D\delta_f}} &= 2l\, \frac{\partial\epsilon}{\partial\delta_f}\, C_{Z_{\alpha_t}} & C_{m_{D\delta_f}} &= 2l\, \frac{\partial\epsilon}{\partial\delta_f}\, C_{m_{\alpha_t}}
\end{aligned}\right\}\tag{8}$$

Note that the moment parameters are identical to the vertical-force parameters except that the subscript m is substituted for Z. The values of $C_{Z_{D\alpha}}$, C_{Z_q}, $C_{m_{D\alpha}}$, and C_{m_q} in the preceding expressions are supplied entirely by the tail, inasmuch as any contributions of the wing-fuselage combination to these values have been neglected. Such contributions could be added to the values in equations (8) if desired.

The terms involving $D\alpha_g$ on the right-hand side of equations (7) result from the lag between the effects of a gust on the wing and the tail when the airplane penetrates a given region of the gust disturbance. The significance of these terms is discussed subsequently.

Calculation of Response to Sinusoidal Gust Disturbances

Transfer functions.- As noted previously, the response of the airplane without an acceleration alleviator to sinusoidal gust disturbances of various frequencies is derived in this section as an illustration of the method to be used with the more complex acceleration-alleviation systems.

In order to determine the steady-state response of the airplane to a sinusoidal gust disturbance, when the controls are fixed, the quantities δ_f and δ_e are set equal to zero. Equations (7) may then be solved algebraically by the method of determinants in terms of α_g for the variables α_o and $D\theta$. The substitution $D = i\omega$ in the resulting expression, which is known as the transfer function, gives a complex quantity, the real and imaginary parts of which represent the components of response in phase and 90° out of phase with an applied disturbance of frequency ω. The derivation of this procedure is given in reference 22 (pp. 175-176).

The transfer functions for α_o and $D\theta$ are as follows:

$$\alpha_o = \alpha_g \frac{\Delta_1}{\Delta_o} \tag{9}$$

$$D\theta = \alpha_g \frac{\Delta_2}{\Delta_o} \tag{10}$$

where

$$\begin{aligned}\Delta_o = {} & D^2\left(-4\mu^2 K_y{}^2 + \mu K_y{}^2 C_{Z_{D\alpha}}\right) + \\ & D\left(\mu C_{m_{D\alpha}} + \mu C_{m_q} + 2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{4} C_{m_{D\alpha}} C_{Z_q} - \frac{1}{4} C_{Z_{D\alpha}} C_{m_q}\right) + \\ & \left(2\mu C_{m_\alpha} + \frac{1}{2} C_{m_\alpha} C_{Z_q} - \frac{1}{2} C_{Z_\alpha} C_{m_q}\right)\end{aligned}$$

$$\Delta_1 = D^2\left(-\mu K_y{}^2 C_{Z_{D\alpha}} + \mu K_y{}^2 C_{Z_q}\right) +$$

$$D\left(-\mu C_{m_{D\alpha}} + \mu C_{m_q} - 2\mu K_y{}^2 C_{Z_\alpha} - \frac{1}{4} C_{m_{D\alpha}} C_{Z_q} + \frac{1}{4} C_{Z_{D\alpha}} C_{m_q}\right) +$$

$$\left(-2\mu C_{m_\alpha} - \frac{1}{2} C_{m_\alpha} C_{Z_q} + \frac{1}{2} C_{Z_\alpha} C_{m_q}\right)$$

$$\Delta_2 = D^2\left(-\mu C_{m_{D\alpha}} + \mu C_{m_q} + \frac{1}{4} C_{m_{D\alpha}} C_{Z_q} - \frac{1}{4} C_{Z_{D\alpha}} C_{m_q}\right) +$$

$$D\left(-2\mu C_{m_\alpha} + \frac{1}{2} C_{m_\alpha} C_{Z_q} - \frac{1}{2} C_{Z_\alpha} C_{m_q}\right)$$

The combination of terms $\frac{1}{4} C_{m_{D\alpha}} C_{Z_q} - \frac{1}{4} C_{Z_{D\alpha}} C_{m_q}$ cancels when only the tail contributions to these stability derivatives (equations (8)) are taken into account.

The normal acceleration may be obtained from equations (9) and (10). The normal acceleration in g units is given by

$$n = \frac{V}{g}\left(\dot{\alpha}_o - \dot{\theta}\right)$$

but

$$\dot{\alpha}_o = \frac{V}{c} D\alpha_o$$

and

$$\dot{\theta} = \frac{V}{c} D\theta$$

hence, in terms of nondimensional quantities,

$$n = \frac{V^2}{gc} D\left(\alpha_o - \theta\right)$$

$$= \frac{D\left(\alpha_o - \theta\right)}{F} \tag{11}$$

where $F = \frac{gc}{V^2}$ (Froude number). From equations (9) and (10) the normal acceleration is

$$n = \frac{\alpha_g}{F} \frac{\Delta_3}{\Delta_o} \tag{12}$$

where

$$\Delta_3 = D^3\left(\mu K_y{}^2 C_{Z_q} - \mu K_y{}^2 C_{Z_{D\alpha}}\right) +$$

$$D^2\left(-2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{2} C_{Z_{D\alpha}} C_{m_q} - \frac{1}{2} C_{m_{D\alpha}} C_{Z_q}\right) +$$

$$D\left(C_{Z_\alpha} C_{m_q} - C_{m_\alpha} C_{Z_q}\right)$$

<u>Relations between gust frequency and wave length</u>.- The substitution $D = i\omega$ in equations (9), (10), and (12) gives the response to a gust of frequency ω, where ω is a circular frequency expressed in radians per chord traveled. The actual frequency in cycles per second is related to ω by the formula

$$N = \frac{\omega V}{2\pi c} \tag{13}$$

The relation between the wave length of the gust disturbance, the flight speed, and the frequency may be determined by inspection of figure 3. The form of the assumed gust disturbance is

$$w_g = w_{g_a} \sin \frac{2\pi x}{\lambda}$$

but $x = Vt$, $\alpha_g = \frac{w_g}{V}$, and $\alpha_{g_a} = \frac{w_{g_a}}{V}$. Hence

$$\alpha_g = \alpha_{g_o} \sin \frac{2\pi Vt}{\lambda}$$

The frequency in cycles per second is therefore

$$N = \frac{V}{\lambda} \tag{14}$$

By comparison of formulas (13) and (14)

$$\omega = \frac{2\pi}{\lambda/c}$$

Convention for phase angles.- Solutions for the phase angles between the variable under consideration and the gust disturbance, obtained from formulas such as (9), (10), and (12), give the phase angle by which the variable leads the disturbance, when positive directions of these quantities are taken the same as in setting up the original equations. In the figures presented herein, the phase angles of pitching velocity and normal acceleration will be plotted exactly as obtained from these formulas. In presenting phase angles for flap and elevator deflections, however, the sign conventions will be changed, where necessary, so that "in phase" represents the condition in which the control moves in the direction that would be intuitively required to offset the effects of the disturbance. For example, when the flaps move up for positive angle of attack, these quantities will be called in phase.

Discussion of Limitations of Theory

Errors in prediction of response of an airplane to gusts may arise because of nonuniformity of gust velocity across the span, unsteady-lift effects, inaccuracy of the approximation to the downwash at the tail, and the effects of airplane flexibility. All these sources of error became more important at high frequencies and must therefore be considered in setting an upper limit to the frequencies at which the analysis would be expected to yield reasonably accurate results. The effect of nonuniformity of gust velocity across the span is believed to be the source of error which first becomes serious as the gust frequency is increased. This effect is therefore considered in some detail.

If turbulence were distributed at random through the atmosphere, the distribution of vertical gust velocity across the wing span would be expected to be similar to the distribution along the flight path. This expectation has been roughly confirmed by studies at the National Advisory Committee for Aeronautics of the gust structure of the atmosphere (reference 23). The assumption of uniform gust velocity across the span therefore is likely to be approximately correct for gusts which have wave lengths long compared with the wing span but not for gusts which have wave lengths of the same order of magnitude as the wing span. Calculation of the response of an airplane to gusts which vary across the span would be a lengthy process because of the large number of combinations of longitudinal and spanwise velocity variations which might occur. For a given maximum gust velocity any nonuniformity of the gust across the span would be expected to reduce the acceleration of the airplane. An approximate idea of the magnitude of this reduction may be obtained by calculating the lift on a fixed wing subjected to an angle of attack distribution which varies sinusoidally across the span. A strip-theory analysis employing this procedure has been used to calculate the lift of wings with taper ratios 1 and 0. The values for any actual wing could be expected to lie between those obtained for taper ratios 1 and 0. For each gust wave length the wing was oriented with respect to the gust in such a way as to produce the greatest lift increment. The results of these calculations, shown in figure 4, are plotted as a function of the quantity ωA, where

$$\omega A = \frac{2\pi}{\lambda/b}$$

Values of ω are also shown for a value of $A = 11$, the aspect ratio of the airplane to be used in subsequent calculations. It may be seen from these curves that a large amount of acceleration alleviation will be provided automatically at values of ω greater than about 0.7 because of nonuniformity of the gusts across the span. This effect probably accounts for the lack of response of an airplane to the high-frequency components of gust disturbances which is illustrated by the measurements of reference 11. An acceleration alleviator therefore would not be required to respond to disturbances of very high frequency.

This reasoning brings out the distinction between the frequency-response requirements for an acceleration alleviator intended for improvement of passenger comfort and a gust alleviator intended to reduce structural loads. The gust alleviator would have to be designed to provide for the chance occurrence of an abrupt gust occurring uniformly across the wing span. Such a condition would be rarely encountered, however, and would therefore be of little concern with regard to passenger comfort.

In the subsequent analysis, no results are given for values of ω greater than 0.7 because the effect of nonuniformity of gusts across the span makes the results based on the assumption of uniform gusts of little interest at higher values of ω.

The errors introduced by unsteady-lift effects and by the approximation to the downwash at the tail are of appreciable magnitude at $\omega = 0.7$ for the airplane without an acceleration alleviator. For cases in which good acceleration alleviation is obtained, however, any errors due to these effects are reduced because the lift increments due to gusts are small and because, as is shown subsequently, downwash changes at the tail must be minimized.

In cases where flaps are used to offset the lift increments due to gusts, unsteady-lift effects due to flap deflection are of interest. Analysis of data on the unsteady-lift characteristics of flaps indicates that the reduction in magnitude of the lift with increasing frequency of flap oscillation, in the frequency range considered, is nearly equal to the corresponding reduction with increasing frequency of a sinusoidal gust. A constant ratio between flap amplitude and gust amplitude may therefore be used to offset the lift due to gusts of any frequency.

The effects of flexibility would be expected to influence considerably the acceleration response of an airplane at frequencies approaching the lowest structural frequency of the airplane. Data for existing transport airplanes indicate that the wing bending frequency at cruising speeds would correspond to a value of ω of about 0.7, the highest frequency for which calculations are presented. In practice, it would probably be desirable for the response of any acceleration-alleviation system to approach zero at frequencies in the neighborhood of the lowest natural structural frequencies of the airplane, in order to avoid the possibility of exciting unstable structural oscillations. The theory as given herein does not take into account such special response characteristics. This theory is used because the simplicity of the analysis allows some general conclusions to be drawn. It should be kept in mind, however, that in the design of an acceleration-alleviating device, the response characteristics of the actual mechanism involved, particularly at high oscillation frequencies, would require more detailed investigation because of the possible influence of the device on flutter. The analysis as presented is reasonably accurate in the lower frequency range which has been shown to be of greatest interest for minimizing airsickness.

CALCULATED RESPONSE OF BASIC AIRPLANE TO GUSTS

The characteristics of the basic airplane for which the response calculations have been made are given in table I. The estimated stability derivatives are given in table II. The same airplane characteristics are assumed in subsequent calculations of the effects of various acceleration-alleviation systems. The values used are fairly representative of a modern transport airplane, with the possible exception that the values of wing loading and inertia are lower than normal. These lower values were selected because they should make the problem of providing smooth flight more difficult. The calculations were made for an airspeed of 200 miles per hour at standard sea-level conditions. The same conditions are used in presenting data for other arrangements throughout the paper.

The phase and amplitude of the normal acceleration and pitching velocity are plotted in figure 5 as functions of the gust frequency for a gust disturbance having an amplitude of 1°. The curves are plotted for three center-of-gravity positions, corresponding to values of static margin in straight flight of 0, 10, and 20 percent of the mean aerodynamic chord. In this and in subsequent figures, accelerations and pitching velocities have been plotted as dimensional quantities in order to aid in visualizing the actual magnitude involved. The results may, of course, be applied more generally to dynamically similar airplanes at other conditions of speed and altitude by use of the original nondimensional equations.

These results show that for gusts of frequency greater than about 0.5 cycle per second the amplitude of acceleration experienced by the airplane is practically the same as the acceleration that would be calculated on the basis of a steady angle-of-attack change equal to the gust amplitude. For gusts of lower frequency, the acceleration decreases because the airplane tends to relieve the acceleration by vertical motion and by pitching into the gusts. There is very little indication of a resonant condition near the natural frequency of the short-period oscillation of the airplane because, for the value of the relative-density factor μ that was assumed, the short-period oscillation is almost critically damped. This condition is typical of transport airplanes.

The pitching velocity caused by gusts varies with frequency in a similar manner to the normal acceleration. At first, it might appear that the pitching velocity should approach zero at high gust frequencies. Such a result would be expected if sinusoidally varying pitching moments of constant amplitude and different frequencies were applied to the airplane. The fact that the pitching velocity approaches a constant value for the case shown in figure 5 results from the assumption that the airplane is traversing a region of the atmosphere containing alternate upward and downward gusts, the velocities of which do not vary with time. The lag between penetration of a given point in the gust profile by the wing and the tail was shown to result in the terms involving $C_{Z_{D\alpha}}$, C_{Z_q}, $C_{m_{D\alpha}}$, and C_{m_q} multiplied by $D\alpha_g$ on the right-hand side of equations (8). Inasmuch as the value of $D\alpha_g$ increases directly with gust frequency, the forces and moments represented by these terms increase directly with gust frequency and thereby offset the tendency of the pitching response of the airplane to decrease with increasing frequency.

Examination of the sketch of a sinusoidal gust (fig. 3) of the type assumed in this analysis shows that, near the points of zero gust velocity, the air has a velocity distribution similar to that which would exist if these portions of the air were rotating as solid bodies, whereas at the points of maximum gust velocity the air is moving vertically up and down. With decreasing wave length of gusts of constant amplitude, the strength of the rotations may be seen to increase. These regions where rotational effects exist may be visualized as applying the previously mentioned aerodynamic forces and moments which increase with increasing gust frequency.

Previous investigators have sometimes considered separately the effects of "vertical gusts" and "rotational gusts." (See references 12 and 24.) The effects of these two types of disturbances could then be combined in the desired proportions. A purely sinusoidal vertical gust would require a vertical oscillation of the entire portion of the atmosphere traversed by the airplane; whereas a purely rotational gust would require a rotational oscillation of the atmosphere about an axis moving with the airplane. These conditions are physically impossible. In fact, any combination of vertical and rotational gusts other than the one assumed in figure 3 would require rapid variations with time of the gust velocities. In practice, the gust velocities in the atmosphere may vary with time, but these variations are believed to occur slowly as compared to the frequency with which the airplane encounters gusts. For this reason, it appears logical to make the assumption that the gust velocities do not vary with time and thereby combine the vertical and rotational gust effects from the start in the manner indicated in figure 3.

CONTROL MOTIONS REQUIRED FOR ELIMINATION OF ACCELERATIONS DUE TO GUSTS

In order to determine a satisfactory method for providing smooth flight through rough air, it is helpful to consider the control motions that would be required to eliminate completely the accelerations caused by sinusoidal gusts of different frequencies. The method of operating the control and the effect of its operation on the stability characteristics of the airplane are neglected for the present. The controls to be considered are the elevator, the wing flaps, and the flaps and elevator in combination. In general, operation of a control surface in such a way as to produce straight-line motion of the center of gravity of the airplane will not eliminate pitching oscillations caused by the gusts. The severity of these pitching motions should be investigated in determining the suitability of different methods of control.

Control by Elevator Alone

Control of the airplane by means of the elevator alone will be considered first. This method is attractive because it requires no additional control surfaces. The elevator motion required to provide zero acceleration of the center of gravity and the corresponding pitching motion of the airplane when it flys through sinusoidal gusts of different frequencies may be determined from the general equations (7) and (8) given in a preceding section. For this purpose, the normal acceleration $D(\alpha_o - \theta)$ is set equal to zero. From this relation

$$D\alpha_o = D\theta \tag{15}$$

and

$$\alpha_o = \frac{D\theta}{D}$$

In addition, δ_f is set equal to zero. When these substitutions are made in equations (7), the resulting expressions may be solved simultaneously for the elevator angle δ_e required for zero acceleration and the resulting pitching velocity $D\theta$. The operational expressions obtained are as follows:

$$\frac{\delta_e}{\alpha_g} = \frac{D^3\left(\mu K_y^2\right)\left(C_{Z_{D\alpha}} - C_{Z_q}\right) + D^2\left(2\mu K_y^2 C_{Z_\alpha}\right) + D\left(C_{m_\alpha}C_{Z_q} - C_{Z_\alpha}C_{m_q}\right)}{D^2\left(-2\mu K_y^2 C_{Z_{\delta_e}}\right) + C_{m_\alpha}C_{Z_{\delta_e}} - C_{Z_\alpha}C_{m_{\delta_e}}} \tag{16}$$

$$\frac{D\theta}{\alpha_g} = \frac{D\left(C_{Z_\alpha}C_{m_{\delta_e}} - C_{m_\alpha}C_{Z_{\delta_e}}\right)}{D^2\left(-2\mu K_y^2 C_{Z_{\delta_e}}\right) + C_{m_\alpha}C_{Z_{\delta_e}} - C_{Z_\alpha}C_{m_{\delta_e}}}$$

In order to show the significance of these results, the phase and amplitude of the elevator motion and pitching velocity per unit amplitude of gust angle of attack have been plotted as functions of gust frequency in figure 6. These calculations were made for the typical transport airplane with the characteristics given in table I. This figure shows that the amplitude of elevator motion required increases almost linearly with frequency and reaches very large values at high frequencies. The variation of elevator angle must lead the variation of gust angle of attack at the wing by a phase angle increasing from 90° at low frequencies to about 160° at high frequencies. Large phase leads such as these may be difficult to attain in practice with an automatic control and cannot be applied by a human pilot, because he has no means of anticipating the gusts. These results indicate the reason for the inability of a human pilot to counteract successfully the effect of gusts by use of the elevator.

The elevator motion required to maintain zero acceleration is independent of the center-of-gravity position of the airplane. Equation (16) may appear to indicate a slight dependence of the elevator angle required on the center-of-gravity position but this small effect disappears if the variation of tail length with center-of-gravity position is taken into account. Physically, the lack of dependence of the elevator motion on center-of-gravity position is caused by the assumption that variations in acceleration and hence in total lift have been reduced to zero. The moments that the elevator must overcome will therefore be the same no matter what the center-of-gravity position. This independence of the center-of-gravity position is an advantage inherently available in any mechanism which operates a perfect acceleration alleviator, inasmuch as no provision needs to be made for changing the characteristics of the mechanism as a function of center-of-gravity position.

A comparison of the values of pitching velocities (fig. 6) with those of the basic airplane (fig. 5) indicates that the pitching velocities are very much greater when the elevator is operated to maintain zero acceleration of the center of gravity. In order to show the effects of these increased pitching velocities on passenger comfort, the values of normal acceleration caused by the pitching oscillations at points 1 chord and 2 chords from the center of gravity of the airplane have been computed. In figure 7 the accelerations caused by pitching when the elevator is operated are compared with the accelerations of the center of gravity of the basic airplane with no acceleration alleviator. At high frequencies the accelerations at points in the cabin located some distance from the center of gravity would be greater when the elevator is used as an acceleration alleviator than they would be in the basic airplane. If it were desired to keep the acceleration at all points in the cabin to some low value, say one-tenth of the value encountered with the basic airplane, then the use of the elevator as an acceleration alleviator would meet this requirement only at gust frequencies corresponding to values of less than about 0.05 radian per chord. For the typical transport airplane under consideration flying at a speed of 200 miles per hour, this frequency would be about 0.26 cycle per second. Most records of the variation of acceleration in rough air show that most bumps occur at frequencies higher than this value. Hence, the elevator control does not appear to be very effective as a means of improving comfort. On the basis of the results of motion-sickness research presented in figure 1, however, elimination of the lower frequency components of motion might have a beneficial effect in reducing airsickness.

The reason for the large pitching motion produced when the elevator is used to reduce the accelerations is that the airplane must be pitched to maintain approximately a constant angle of attack with respect to the air stream during passage through the gusts. The change in angle of attack of the airplane is given by the formula

$$\alpha = \alpha_o + \alpha_g$$

If the angle of attack is constant,

$$D\alpha_o + D\alpha_g = 0$$

From formula (15), the condition for zero acceleration of the center of gravity is

$$D\alpha_o = D\theta$$

Hence the value of pitching velocity required to maintain the angle of attack constant is

$$D\theta = -D\alpha_g$$

The pitching velocity which would result as a function of frequency in accordance with this assumption is shown as a dashed line in figure 6. The departure of the actual pitching velocity from this relation at higher values of frequency is caused by the lift on the tail, which acts in a direction to reduce the accelerations caused by the gusts. As a result less pitching of the entire airplane is required to maintain constant total lift than would be required to maintain a constant angle of attack.

If a given airplane traverses at various speeds a region containing a given pattern of rough air, the pitching velocities produced when the elevator is used as an acceleration alleviator increase directly with the airspeed. The suitability of the elevator as a control for reducing accelerations therefore becomes progressively poorer with increasing airspeed.

Control by Flaps Alone

The use of flaps on the wing to offset the lift increments due to gusts might be expected to avoid the excessive pitching motions that were obtained when the elevator was used because the flaps produce lift directly without the necessity of pitching the airplane. In order to investigate the flap motion required to provide zero acceleration of the center of gravity, and the corresponding pitching motion of the airplane when it flies through sinusoidal gusts of different frequencies, equations (7) and (8) may be used. These equations may be solved simultaneously for the flap angle δ_f required for zero acceleration and the resulting pitching velocity $D\theta$ by setting the normal acceleration $D\left(\alpha_o - \theta\right)$ and the elevator angle δ_e equal to zero. The operational expressions obtained are as follows:

$$\frac{\delta_f}{\alpha_g} = \frac{\Delta_5}{\Delta_4}$$

$$\frac{D\theta}{\alpha_g} = \frac{\Delta_6}{\Delta_4} \tag{17}$$

where

$$\Delta_4 = D^3\left(-\mu K_y{}^2 C_{Z_{D\delta_f}}\right) + D^2\left(-2\mu K_y{}^2 C_{Z_{\delta_f}}\right) +$$

$$D\left[\frac{1}{2} C_{m_\alpha} C_{Z_{D\delta_f}} - \frac{1}{2} C_{Z_\alpha} C_{m_{D\delta_f}} + \frac{1}{2}\left(C_{m_{D\alpha}} + C_{m_q}\right) C_{Z_{\delta_f}} - \frac{1}{2}\left(C_{Z_{D\alpha}} + C_{Z_q}\right) C_{m_{\delta_f}}\right] +$$

$$C_{m_\alpha} C_{Z_{\delta_f}} - C_{Z_\alpha} C_{m_{\delta_f}}$$

$$\Delta_5 = D^3\left(\mu K_y{}^2\right)\left(C_{Z_{D\alpha}} - C_{Z_q}\right) + D^2\left(2\mu K_y{}^2 C_{Z_\alpha}\right) + D\left(C_{m_\alpha} C_{Z_q} - C_{Z_\alpha} C_{m_q}\right)$$

$$\Delta_6 = D^2\left[\frac{1}{2}\left(C_{Z_{D\alpha}} - C_{Z_q}\right) C_{m_{\delta_f}} - \frac{1}{2}\left(C_{m_{D\alpha}} - C_{m_q}\right) C_{Z_{\delta_f}} + \frac{1}{2} C_{Z_\alpha} C_{m_{D\delta_f}} - \frac{1}{2} C_{m_\alpha} C_{Z_{D\delta_f}}\right] +$$

$$D\left(C_{Z_\alpha} C_{m_{\delta_f}} - C_{m_\alpha} C_{Z_{\delta_f}}\right)$$

Results have been calculated for the typical transport airplane with the characteristics given in table I. Two sets of assumptions, designated cases A and B, were made for the flap characteristics.

The values for case A were selected as typical of those that might be expected for a transport airplane using the landing flaps as an acceleration alleviating control. In this case, downward deflection of the flaps produces a negative pitching moment on the wing itself but a positive moment on the airplane, for the range of center-of-gravity positions considered. This nose-up moment due to flap deflection is typical of many existing transport airplanes. In order to cover a range of conditions, however, the value of $\frac{\partial\epsilon}{\partial\delta_f}$ was arbitrarily reduced for case B to give an over-all value of $C_{m_{\delta_f}}$ of zero at a static margin of 15 percent of the mean aerodynamic chord. The values for the flap characteristics are as follows:

	Case A	Case B
$\left(C_{Z_{\delta_f}}\right)_w$	-1.40	-1.40
$\left(C_{m_{\delta_f}}\right)_w$	-0.338	-0.338
$\frac{\partial\epsilon}{\partial\delta_f}$	0.271	0.144

The values of $C_{m_{\delta_f}}$ for the whole airplane obtained with various center-of-gravity positions for each of these two cases are as follows:

Center-of-gravity position, percent M.A.C.	0.20	0.30	0.40
Static margin, percent M.A.C.	0.20	0.10	0
$C_{m_{\delta_f}}$, for case A	0.232	0.372	0.512
$C_{m_{\delta_f}}$, for case B	-0.070	0.070	0.210

The results, given in figure 8, show, surprisingly, that very large amplitude pitching motions occur even at fairly low gust frequencies when the flap assumed in case A is used as an acceleration alleviator. The pitching velocities are even larger than those obtained by using elevator control. In addition, much larger flap motions are required than those which would offset the lift increment on the wing if the airplane did not pitch. These results indicate that an acceleration alleviator which employs the landing flaps to oppose the lift increment due to gusts, without regard for the pitching moments due to the flaps, is not likely to be successful.

The results obtained with the flap of case B are somewhat more promising, though the pitching velocities are greater than those with elevator control for gusts in the low-frequency range. The pitching velocities at high gust frequencies are about half of those obtained with elevator control. The discussion given in the preceding section indicates that such pitching velocities would still be much too large to be satisfactory. Comparison of cases A and B shows a trend toward more satisfactory results as the downwash due to the flaps $\frac{\partial\epsilon}{\partial\delta_f}$ is reduced. Reducing this value to the point where the value of $C_{m_{\delta_f}}$ for the airplane is about zero does not appear to be sufficient, however.

The phase angles between flap deflection and angle of attack plotted in figure 8 show that, at low gust frequencies, the flap deflection should lead the gust by 90°, whereas, at high gust frequencies, it should lag by a small amount. These requirements would be easier to realize in practice than the large phase leads encountered with elevator control.

Control by Use of Flaps and Elevator

Because of the apparently unfavorable results obtained with either the flaps alone or elevator alone as acceleration alleviators, calculations were made for the case of combined flap and elevator control. By operating these two controls in the correct manner during flight through gusts, both the acceleration and the pitching velocity may be reduced to zero.

In order to determine the flap and elevator motion required for zero acceleration and pitching velocity in flight through sinusoidal gusts of different frequencies, equations (7) and (8) may be used. The acceleration $D(\alpha_o - \theta)$ and the pitching velocity $D\theta$ are set equal to zero and the resulting equations are solved simultaneously for the flap angle and elevator angle. The operational expressions obtained are as follows:

$$\frac{\delta_e}{\alpha_g} = \frac{D\left[\frac{1}{2}\left(C_{Z_{D\alpha}} - C_{Z_q}\right)C_{m_{\delta_f}} - \frac{1}{2}\left(C_{m_{D\alpha}} - C_{m_q}\right)C_{Z_{\delta_f}} + \frac{1}{2}\,C_{Z_\alpha}C_{m_{D\delta_f}} - \frac{1}{2}\,C_{m_\alpha}C_{Z_{D\delta_f}}\right] + C_{Z_\alpha}C_{m_{\delta_f}} - C_{m_\alpha}C_{Z_{\delta_f}}}{C_{m_{\delta_e}}C_{Z_{\delta_f}} - C_{Z_{\delta_e}}C_{m_{\delta_f}}} \tag{18}$$

$$\frac{\delta_f}{\alpha_g} = \frac{C_{Z_{\delta_e}}C_{m_\alpha} - C_{m_{\delta_e}}C_{Z_\alpha}}{C_{m_{\delta_e}}C_{Z_{\delta_f}} - C_{Z_{\delta_e}}C_{m_{\delta_f}}} \tag{19}$$

The phase and amplitude of the flap and elevator motions required per unit amplitude of gust angle of attack have been plotted in figure 9 as functions of gust frequency. The flap characteristics were assumed to be those given previously as cases A and B.

The phase and amplitude of the flap motion are constant. As would be expected, the flaps should move in phase with the gusts. In this case, an automatic control would be arranged to move the elevator up when an upward gust hits the wing. Figure 9 shows that the elevator, like the flaps, should move in phase with the gust at low gust frequencies, but should lag behind the gust by an increasing phase angle and move with increasing amplitudes at higher gust frequencies. The flap and elevator motions given by equations (18) and (19) may be interpreted physically as those required to maintain the airplane in equilibrium at all times during passage through gusts.

The use of combined flap and elevator control to offset the effects of gusts offers a possible method of providing smooth flight. The design of a practical mechanism using this method of control appears complicated, however, because of the different amplitude and phase relationships required for the flap and elevator motion at different frequencies. For this reason, an attempt has been made to determine the conditions under which smooth flight could be obtained by use of the flaps alone. In order to accomplish this result, the large pitching motions accompanying flap motion in the examples worked out previously must be eliminated.

CHARACTERISTICS REQUIRED OF FLAPS USED AS ACCELERATION ALLEVIATORS

The flap characteristics needed to eliminate accelerations and pitching motions due to gusts are now derived. Certain values of pitching moment due to flap deflection are shown to be required. In practice, the desired values might not be obtained with trailing-edge flaps, but these values could be obtained by linking the elevator, or a portion of the elevator, to deflect in phase with the flaps. Arrangements in which the flaps and elevator are linked to deflect together will therefore be considered as included in this discussion. The characteristics of the combination may be considered as flap characteristics for purposes of analysis.

The conditions for elimination of the pitching motion that occurs when flaps are used as acceleration alleviators may be derived by setting the numerator of the formula for the pitching velocity (equation (17)) equal to zero. Alternatively, a solution may be obtained for the conditions under which zero elevator motion is required to provide smooth flight by setting the numerator of the formula for the elevator angle (equation (18)) equal to zero. It is seen that these two expressions are exactly the same, and in either case the result is

$$D\left[\frac{1}{2}\left(C_{Z_{D\alpha}} - C_{Z_q}\right)C_{m_{\delta_f}} - \frac{1}{2}\left(C_{m_{D\alpha}} - C_{m_q}\right)C_{Z_{\delta_f}} + \frac{1}{2}\, C_{Z_\alpha}C_{m_{D\delta_f}} - \frac{1}{2}\, C_{m_\alpha}C_{Z_{D\delta_f}}\right] +$$

$$C_{Z_\alpha}C_{m_{\delta_f}} - C_{m_\alpha}C_{Z_{\delta_f}} = 0 \tag{20}$$

In order for this expression to equal zero at all gust frequencies, both terms must equal zero separately. Thus two simultaneous equations are provided which allow the determination of two parameters.

The two quantities which may be varied by changing the design of the flaps, without too greatly affecting their lift capabilities, are the variation of flap pitching moment with deflection $\left(C_{m_{\delta_f}}\right)_w$ and downwash due to the flap $\frac{\partial\epsilon}{\partial\delta_f}$. These quantities were therefore chosen as the independent variables. The parameters in equation (20) are expanded in terms of these and other basic parameters in accordance with formulas (8). The resulting expressions may be simplified and solved for $\left(C_{m_{\delta_f}}\right)_w$ and $\frac{\partial\epsilon}{\partial\delta_f}$ to yield

$$\left(C_{m_{\delta_f}}\right)_w = \frac{C_{m_{\alpha_w}}\left(C_{Z_{\delta_f}}\right)_w}{C_{Z_{\alpha_w}}} \tag{21}$$

$$\frac{\partial\epsilon}{\partial\delta_f} = \frac{-\left(C_{Z_{\delta_f}}\right)_w}{C_{Z_{\alpha_w}}}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) \tag{22}$$

When these conditions are satisfied, the flaps will act as perfect acceleration alleviators; that is, operation of the flaps to offset the accelerations produced by gusts will result in no pitching motions. The flap deflection required under these conditions may be shown to be in phase with the angle of attack due to the gusts and to have an amplitude ratio independent of frequency.

Physically, these results may be interpreted as follows: When the first condition is satisfied, deflection of the flaps to offset the wing lift due to a gust will produce a pitching moment about the center of gravity which just offsets the pitching moment of the wing due to angle of attack. Thus the wing contributes no lift or pitching moment. When the second condition is satisfied, the downwash due to flap deflection just offsets the angle of attack at the tail due to the gust, which equals $\alpha_g\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)$. Thus the tail also contributes no lift or pitching moment.

These characteristics are not obtained with normal flap arrangements. The first condition requires flaps which produce zero pitching moment about the wing quarter-chord point. The second condition requires flaps with a negative value of $\frac{\partial\epsilon}{\partial\delta_f}$, that is, flaps which produce downwash in the region of the tail when they are deflected up. Characteristics equivalent to those desired may possibly be obtained, however, by combining flap and elevator deflections and by making certain modifications to the usual flap arrangement. The pitching moment of the flaps may be adjusted to any desired value with little effect on their lift capabilities by linking the flaps directly to the elevator or to a portion of the elevator. A reversal in the direction of downwash at the tail caused by flap deflection might be obtained by linking a portion of each flap near the wing root to deflect in the opposite direction from the rest of the flap. Whether the desired downwash value could be realized on any actual airplane configuration would have to be determined by wind-tunnel tests. The use of such an arrangement would reduce the lift increment that could be produced by the flaps at their maximum usable deflection. By use of suitable gearing between the gust sensing device and the flaps, however, the lift increments due to gusts could still be completely offset up to some value of gust amplitude. Beyond this amplitude, the additional lift due to the gust would be the same as for the basic airplane. Even though the flap effectiveness were considerably reduced, therefore, the effects of the frequently encountered small-amplitude disturbances, which are most important with regard to passenger comfort, could be completely offset and the frequency of exceeding accelerations beyond any given amplitude would be reduced. The adverse effect of such an arrangement on the lift capabilities of the flaps could be reduced by using flaps that cover a large portion of the span.

EFFECTIVENESS OF POSSIBLE FLAP CONTROL SYSTEMS AS ACCELERATION ALLEVIATORS

The action of any practical acceleration alleviator is not likely to be exactly in accordance with the ideal behavior calculated previously. For this reason, calculations must be made to determine the characteristics of any actual system. Two main types of systems have been analyzed, one in which the flaps are operated in accordance with the indications of a vane sensitive to angle-of-attack changes and the other in which the flaps are operated in accordance with the indications of an accelerometer. This section deals with the effectiveness of these systems in reducing the response of an airplane to gusts. The problems of stability and control involved in the use of these systems are considered in the following section.

Vane-Controlled Acceleration Alleviator

Method of analysis.- A sketch of the system under consideration is shown in figure 10. The flaps are operated by a servomechanism in accordance with the indications of a freely floating vane mounted on a boom ahead of the nose of the fuselage. The linkage to the pilot's control may be disregarded for the present. In the operation of this system, an upward gust would cause the vane to deflect upward. The servomechanism would then deflect the flaps upward to counteract the lift increment due to the gust. The time lag in the operation of the servomechanism presumably could be adjusted to compensate for the time lag between penetration of the gust by the vane and by the wing.

In the nondimensional notation discussed previously, the expression for the vane deflection is as follows:

$$\delta_v = -\alpha_o - \alpha_g e^{l_n D} + D\theta \ l_n \qquad (23)$$

The upwash ahead of the wing, which amounts to only a few percent of the angle of attack for ordinary vane locations, has been neglected in setting up the expression for the vane deflection.

The frequency-response characteristics of the servomechanism determine the relation between flap motion and vane motion. The assumption is made that the servomechanism produces a constant ratio between flap and vane motion and introduces a constant time lag, independent of frequency. This behavior is a good approximation to the measured characteristics of actual servomechanisms which are considered suitable for this application at frequencies well below the natural frequencies of the servomechanisms. The flap motion is then determined by the relation

$$\delta_f = K\delta_v e^{-\tau D} \tag{24}$$

If equation (23) is substituted in equation (24), the flap motion is given by the relation

$$\delta_f = K\left[-\alpha_o e^{-\tau D} - \alpha_g e^{\left(l_n - \tau\right)D} + D\theta \ l_n e^{-\tau D}\right]$$

This equation may be substituted in equation (5) for α_t. The exponential terms in the expressions for δ_f and α_t are again approximated by the first two terms of their series expansions. Thus the quantities α_w, α_t, and δ_f required for substitution in the original equations of motion (equations (2)) are all expressed in terms of the variables α_o, $D\theta$, and α_g. (The quantity δ_e is set equal to zero.) The equations may then be solved as before for α_o and $D\theta$ in terms of α_g. The resulting acceleration n may be determined in accordance with equation (11). The transfer functions for $D\theta$ and n in terms of α_g are as follows:

$$\frac{n}{\alpha_g} = \frac{1}{F}\frac{\Delta_8}{\Delta_7} \tag{25}$$

$$\frac{D\theta}{\alpha_g} = \frac{\Delta_9}{\Delta_7} \tag{26}$$

where

$$\Delta_7 = D^2\left(-4\mu^2 K_y{}^2 + \mu K_y{}^2 C_{Z_{D\alpha}} + \frac{1}{2}\mu C_{m_{Dq}} + \frac{1}{8} C_{Z_{Dq}} C_{m_{D\alpha}} - \frac{1}{8} C_{m_{Dq}} C_{Z_{D\alpha}}\right) +$$

$$D\left(\mu C_{m_{D\alpha}} + \mu C_{m_q} + 2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{4} C_{m_{D\alpha}} C_{Z_q} - \frac{1}{4} C_{Z_{D\alpha}} C_{m_q} +\right.$$

$$\left.\frac{1}{4} C_{m_\alpha} C_{Z_{Dq}} - \frac{1}{4} C_{Z_\alpha} C_{m_{Dq}}\right) + 2\mu C_{m_\alpha} + \frac{1}{2} C_{m_\alpha} C_{Z_q} - \frac{1}{2} C_{Z_\alpha} C_{m_q}$$

$$\Delta_8 = D^3\left(\mu K_y{}^2 C_{Z_q} - \mu K_y{}^2 C_{Z_{D\alpha}} + \frac{1}{8} C_{Z_{D\alpha}} C_{m_{Dq}} - \frac{1}{8} C_{m_{D\alpha}} C_{Z_{Dq}} + \frac{1}{8} C_{Z_{Dq}} C_{m_q} - \frac{1}{8} C_{m_{Dq}} C_{Z_q}\right) +$$

$$D^2\left(-2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{2} C_{Z_{D\alpha}} C_{m_q} - \frac{1}{2} C_{m_{D\alpha}} C_{Z_q} + \frac{1}{4} C_{m_{Dq}} C_{Z_\alpha} - \frac{1}{4} C_{Z_{Dq}} C_{m_\alpha}\right) +$$

$$D\left(C_{Z_\alpha} C_{m_q} - C_{m_\alpha} C_{Z_q}\right)$$

$$\Delta_9 = D^2\left(-\mu C_{m_{D\alpha}} + \mu C_{m_q} + \frac{1}{4} C_{Z_q} C_{m_{D\alpha}} - \frac{1}{4} C_{Z_{D\alpha}} C_{m_q}\right) +$$

$$D\left(-2\mu C_{m_\alpha} + \frac{1}{2} C_{m_\alpha} C_{Z_q} - \frac{1}{2} C_{Z_\alpha} C_{m_q}\right)$$

It will be noted that these expressions are similar to equations (10) and (12) for the response of the basic airplane with the exception that terms involving $C_{Z_{Dq}}$ and $C_{m_{Dq}}$ are added. The various derivatives differ from those for the basic airplane, however, and are defined as follows:

$$\left.\begin{aligned}
C_{Z_\alpha} &= C_{Z_{\alpha_w}} + C_{Z_{\alpha_t}}\left(1 - \frac{\partial \epsilon}{\partial \alpha} + K \frac{\partial \epsilon}{\partial \delta_f}\right) - \left(C_{Z_{\delta_f}}\right)_w K \\
C_{Z_{D\alpha}} &= -2C_{Z_{\alpha_t}}\left[-l \frac{\partial \epsilon}{\partial \alpha} + K \frac{\partial \epsilon}{\partial \delta_f}(\tau + l)\right] + 2\left(C_{Z_{\delta_f}}\right)_w \tau K \\
C_{Z_q} &= -2C_{Z_{\alpha_t}}\left(K l_n \frac{\partial \epsilon}{\partial \delta_f} - l\right) + 2\left(C_{Z_{\delta_f}}\right)_w K l_n \\
C_{Z_{Dq}} &= 4C_{Z_{\alpha_t}} K l_n \frac{\partial \epsilon}{\partial \delta_f}(\tau + l) - 4\left(C_{Z_{\delta_f}}\right)_w K l_n \tau
\end{aligned}\right\} \qquad (27a)$$

$$\left.\begin{aligned}
C_{m_\alpha} &= C_{m_{\alpha_w}} + C_{m_{\alpha_t}}\left(1 - \frac{\partial\epsilon}{\partial\alpha} + K\frac{\partial\epsilon}{\partial\delta_f}\right) - \left(C_{m_{\delta_f}}\right)_w K \\
C_{m_{D\alpha}} &= -2C_{m_{\alpha_t}}\left[-l\frac{\partial\epsilon}{\partial\alpha} + K\frac{\partial\epsilon}{\partial\delta_f}(\tau + l)\right] + 2\left(C_{m_{\delta_f}}\right)_w \tau K \\
C_{m_q} &= -2C_{m_{\alpha_t}}\left(Kl_n\frac{\partial\epsilon}{\partial\delta_f} - l\right) + 2\left(C_{m_{\delta_f}}\right)_w Kl_n \\
C_{m_{Dq}} &= 4C_{m_{\alpha_t}}Kl_n\frac{\partial\epsilon}{\partial\delta_f}(\tau + l) - 4\left(C_{m_{\delta_f}}\right)_w Kl_n\tau
\end{aligned}\right\} \qquad (27b)$$

With the exception of the added derivatives $C_{Z_{Dq}}$ and $C_{m_{Dq}}$, therefore, the effect of the vane control is simply to cause changes in the stability derivatives of the basic airplane. In particular, it should be noted that the effective variation of vertical-force coefficient with angle of attack C_{Z_α} may be reduced to zero.

Effectiveness of vane-controlled acceleration alleviators with various flap characteristics.- Results presented in the section on control motions required for elimination of accelerations due to gusts showed that flaps with conventional characteristics produced excessive pitching motions of the airplane. It was desired to investigate whether these undesirable characteristics would still be obtained when the flaps were operated in accordance with the indication of a vane. The results might be expected to be somewhat different in this case because the flap motion will be a function of angle of attack and pitching velocity and will not necessarily correspond to that determined previously to provide zero acceleration. Calculations have therefore been made of the effectiveness of various vane-controlled acceleration alleviators in reducing the response to gusts of the airplane described in table I.

The assumed flap characteristics are the same as those listed previously in the section entitled "Control by Flaps Alone" as cases A and B. The only additional quantities required are the values of K and τ. The value of $\tau = 2.22$ was chosen for these cases as equal to the nondimensional time for the gust to travel from the vane to the center of gravity. This value corresponds, for the stated conditions, to a time lag of 0.068 second, a value in the range attainable with suitable booster mechanisms.

An effort was made to select a value of K which would provide the greatest acceleration alleviation. It may be shown that the value of the coefficient of D in the numerator of equation (26) is a function of K alone. If this term is set equal to zero, the acceleration of the airplane will be zero for small values of ω. This relation was used to select values of K for initial calculations.

The two cases studies are summarized as cases 1 and 2 in table III. (For later reference, the various cases studied are listed in this table in the order in which they are discussed.) The results are presented in figures 11 and 12. Comparison of these results with figure 5 indicates that the systems investigated did not reduce the accelerations much below those of the basic airplane and that these systems increased the pitching velocity due to gusts. Additional calculations with larger values of K showed even more unfavorable characteristics. These results appear to confirm the conclusion that flaps with conventional pitching moment and downwash characteristics will be unsuitable for use as acceleration alleviators.

An effort was made to determine whether more favorable characteristics could be obtained with conventional flap arrangements by modifying the characteristics of the servomechanism. As mentioned previously, the value of the coefficient of D in the numerator of equation (26) is a function of K. Furthermore, the value of the coefficient of D^2 is a function of K and τ. By suitable choice of the values of K and τ, therefore, both of these coefficients may be made to equal zero. Use of these values should provide a somewhat larger range of frequencies over which the acceleration due to gusts will be small. Use of this procedure showed, however, that negative values of τ would be required in conjunction with the flap characteristics assumed previously in order to satisfy these relations. Such negative values are not physically realizable. If the downwash due to flap deflection $\frac{\partial \epsilon}{\partial \delta_f}$ were reduced, however, more reasonable positive values of τ would be required. For this reason calculations were made with the value of $\frac{\partial \epsilon}{\partial \delta_f} = 0$ and with the values of K and τ determined to make the coefficients of D and D^2 in the numerator of equation (26) equal zero. The characteristics assumed are given as case 3 in table III. The results are presented in figure 13. Comparison of these results with figures 5 and 12 shows that in this case considerable reduction in acceleration has been obtained at low and moderate gust frequencies and that the pitching velocities are not much greater than those of the basic airplane. It will be shown later that this particular arrangement is unsatisfactory with regard to stability considerations. Nevertheless, reduction of the downwash due to flaps is shown to be very beneficial for acceleration alleviation.

Effectiveness of vane-controlled acceleration alleviators using optimum flap characteristics.- Values have been derived previously for the flap characteristics required to provide perfect acceleration alleviation. Flaps with these characteristics will also operate perfectly in conjunction with a vane control provided that the characteristics of the servomechanism are as follows:

$$\left.\begin{aligned} K &= \frac{C_{Z_{\alpha_w}}}{\left(C_{Z_{\delta_f}}\right)_w} \\ \tau &= l_n \end{aligned}\right\} \tag{28}$$

If these values together with those of equations (21) and (22) are substituted in the equations for normal acceleration and pitching velocity (equations (25) and (26)), the normal acceleration and pitching velocity due to sinusoidal gusts may be shown to equal zero at all gust frequencies. This result may also be shown more simply from physical considerations. Previous discussion pointed out that with these flap characteristics the ratio of flap deflection to gust angle of attack would be independent of frequency and that the flap motion would be in phase with the variations of gust angle of attack at the wing. The use of a vane control in conjunction with a servomechanism which provides a time lag equal to the time required for the gust to travel from the vane to the wing will automatically provide the flap motion required. The value of K given in formula (28) is just that required for the flap deflection to offset the lift increment on the wing due to an angle-of-attack change.

The values for the arrangement which would provide perfect acceleration alleviation are listed as case 4 in table III. The results are not presented in a figure inasmuch as the acceleration and pitching response are zero at all frequencies.

In the previous section possible methods for obtaining such flap characteristics were discussed. The stability and control characteristics of the airplane with such a system are considered in detail in a subsequent section. From the discussion of the method of obtaining perfect acceleration alleviation, however, it may be seen that flaps providing this characteristic, when used with a vane control, will cause the variation of pitching moment with angle of attack C_{m_α} to equal zero. In order to obtain satisfactory stability, C_{m_α} should be negative. An additional case was therefore investigated in which the values of the flap parameters were selected to approximate those for perfect acceleration alleviation but to provide a small negative value of C_{m_α} of -0.20. The values are listed as case 5 in table III and the results are presented in figure 14. Comparison of these results with figure 5 shows that the accelerations due to gusts have been reduced to less than one-fifth of those for the basic airplane at all gust frequencies. The pitching velocities remain small and about equal to those of the basic airplane.

Effect of airspeed.- Changes in airspeed affect the percent reduction in acceleration produced by a vane-controlled acceleration alleviator only as a result of the fact that the time lag in the booster is ordinarily independent of airspeed and the lag τ, expressed in chords, therefore increases in proportion to the speed. If the time lag in the booster is adjusted for the cruising condition, it will therefore be smaller than the optimum value in low-speed flight. This effect is not believed to be serious, however, because the accelerations experienced by a given airplane in flight through a given region of rough air decrease directly with decreases in airspeed.

Accelerometer-Controlled Acceleration Alleviator

Method of analysis.- A sketch of the system under consideration is shown in figure 15. In the operation of this system, an upward acceleration would cause a proportional upward deflection of the flaps. Effects of time lag in the flap operation will be neglected for the present, for the following reason: The accelerometer control with zero lag causes the flaps to move in the same phase relationship to the gust as the vane control with lag equal to the time required for the gust to travel from the vane to the wing. A comparison made on this basis therefore provides a direct indication of the difference in the effectiveness of the two methods under optimum phase conditions. In practice, some lag would exist in the accelerometer control. The effect of such lag will be considered later.

The flap deflection is given by the following expression:

$$\delta_f = KD\left(\alpha_o - \theta\right) \qquad (29)$$

This expression may be substituted into equations (7) and the resulting expressions solved as before for α_o and $D\theta$ in terms of α_g. The resulting acceleration n may be determined in accordance with equation (11). The expressions for $D\theta$ and n are as follows:

$$\frac{n}{\alpha_g} = \frac{1}{F}\frac{\Delta_{11}}{\Delta_{10}} \tag{30}$$

$$\frac{D\theta}{\alpha_g} = \frac{\Delta_{12}}{\Delta_{10}} \tag{31}$$

where

$$\Delta_{10} = D^3\left(\frac{1}{2}\mu K_y{}^2 C_{Z_{D^2\alpha}}{}'\right) + D^2\left(-4\mu^2 K_y{}^2 + \mu K_y{}^2 C_{Z_{D\alpha}}{}' + \frac{1}{2}\mu C_{m_{Dq}}{}' + \frac{1}{2}\mu C_{m_{D^2\alpha}}{}' + \right.$$

$$\left.\frac{1}{8} C_{Z_{Dq}}{}' C_{m_{D\alpha}}{}' - \frac{1}{8} C_{m_{Dq}}{}' C_{Z_{D\alpha}}{}' + \frac{1}{8} C_{Z_q}{}' C_{m_{D^2\alpha}}{}' - \frac{1}{8} C_{m_q}{}' C_{Z_{D^2\alpha}}{}'\right) +$$

$$D\left(\mu C_{m_{D\alpha}}{}' + \mu C_{m_q}{}' + 2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{4} C_{m_{D\alpha}}{}' C_{Z_q}{}' - \frac{1}{4} C_{Z_{D\alpha}}{}' C_{m_q}{}' + \right.$$

$$\left.\frac{1}{4} C_{m_\alpha} C_{Z_{Dq}}{}' - \frac{1}{4} C_{Z_\alpha} C_{m_{Dq}}{}'\right) + 2\mu C_{m_\alpha} + \frac{1}{2} C_{m_\alpha} C_{Z_q}{}' - \frac{1}{2} C_{Z_\alpha} C_{m_q}{}'$$

$$\Delta_{11} = D^3\left(\mu K_y{}^2 C_{Z_q} - \mu K_y{}^2 C_{Z_{D\alpha}}\right) + D^2\left(-2\mu K_y{}^2 C_{Z_\alpha} + \frac{1}{2} C_{Z_{D\alpha}} C_{m_q} - \frac{1}{2} C_{m_{D\alpha}} C_{Z_q}\right) +$$

$$D\left(C_{Z_\alpha} C_{m_q} - C_{m_\alpha} C_{Z_q}\right)$$

$$\Delta_{12} = D^2\left[-\mu C_{m_{D\alpha}} + \mu C_{m_q} + \frac{1}{4} C_{Z_{D\alpha}}{}'\left(C_{m_{D\alpha}} - C_{m_q}\right) - \frac{1}{4} C_{m_{D\alpha}}{}'\left(C_{Z_{D\alpha}} - C_{Z_q}\right) + \right.$$

$$\left.\frac{1}{4} C_{Z_{D^2\alpha}}{}' C_{m_\alpha} - \frac{1}{4} C_{m_{D^2\alpha}}{}' C_{Z_\alpha}\right] + D\left[-2\mu C_{m_\alpha} + \frac{1}{2} C_{Z_\alpha}\left(C_{m_{D\alpha}} - C_{m_q}\right) - \right.$$

$$\left.\frac{1}{2} C_{m_\alpha}\left(C_{Z_{D\alpha}} - C_{Z_q}\right) + \frac{1}{2} C_{Z_{D\alpha}}{}' C_{m_\alpha} - \frac{1}{2} C_{m_{D\alpha}}{}' C_{Z_\alpha}\right]$$

In contrast to the solution given previously for the vane control, substitution of the expressions for flap deflection does not change the terms on the right-hand side of equations (7). These terms represent the forces and moments applied by the gusts to the basic airplane. The accelerometer control changes the terms on the left-hand side of these equations, which determine the response of an airplane to a disturbance. In particular, the control increases the coefficient $D\left(\alpha_o - \theta\right)$. This change is equivalent to increasing the effective inertia of the airplane resisting vertical accelerations. In order to distinguish between the derivatives of the basic airplane and the derivatives changed by the accelerometer control, a prime has been added to those which are changed by the accelerometer control. Derivatives of the basic airplane are left unprimed and are given in formulas (8).

The values of the derivatives changed by the control are as follows:

$$\left.\begin{aligned}
C_{Z_{D\alpha}}' &= 2\left(C_{Z_{D\alpha}} + KC_{Z_{\delta_f}}\right) \\
C_{Z_{D^2\alpha}}' &= 4KC_{Z_{D\delta_f}} \\
C_{Z_q}' &= 2\left(C_{Z_q} - KC_{Z_{\delta_f}}\right) \\
C_{Z_{Dq}}' &= -4KC_{Z_{D\delta_f}} \\
C_{m_{D\alpha}}' &= 2\left(C_{m_{D\alpha}} + KC_{m_{\delta_f}}\right) \\
C_{m_{D^2\alpha}}' &= 4KC_{m_{D\delta_f}} \\
C_{m_q}' &= 2\left(C_{m_q} - KC_{m_{\delta_f}}\right) \\
C_{m_{Dq}}' &= -4KC_{m_{D\delta_f}}
\end{aligned}\right\} \qquad (32)$$

Effectiveness of accelerometer-controlled acceleration alleviators.- Calculations have been made of the effectiveness of various accelerometer-controlled acceleration alleviators in reducing the response of an airplane to gusts. The airplane characteristics assumed were again those given in table I. Three sets of values for the flap parameters were assumed, corresponding to those used in the calculations for the vane control for cases 2, 4, and 5. The only remaining parameter is the gearing constant K relating the flap deflection and the normal acceleration (formula (29)). Physical reasoning indicates that this value should be large enough to increase the effective inertia resisting vertical motions by a fairly large factor. The values selected increase this inertia by a factor of roughly 4. The values investigated are listed as cases 6, 7, and 8 in table III and the results are plotted in figures 16, 17, and 18.

Comparison of the results of figure 16 with those of figures 5 and 12 shows that the accelerometer control, like the vane control, was not particularly effective in reducing the accelerations of the airplane due to gusts when it was used in conjunction with flaps producing appreciable downwash. In addition, this accelerometer control caused very large pitching velocities compared with those of the basic airplane. Comparison of the results of figure 17 or 18 with those of figures 5 and 14 shows that the accelerometer control was very satisfactory, however, when used in conjunction with flaps producing small downwash. The results in this case were very similar to those obtained with the vane control, except that the vane control appears to be somewhat more effective in reducing accelerations at low frequencies.

Effect of time lag.- In comparing the results of the vane and accelerometer controls (fig. 14 with figs. 17 and 18), it should be noted that the accelerations obtained with the accelerometer control might be further reduced by increasing the value of K, whereas the value of K for the vane control is probably close to the optimum. The vane control, however, already incorporated a time lag in its operation, whereas the accelerometer control was assumed to operate with no time lag. The effect of a time lag in the operation of the accelerometer control was therefore investigated in supplementary calculations. A lag of 2 chords (corresponding to a time lag of 0.061 sec at 200 mph) was found to have little effect on the results obtained for case 8, except that the accelerations at the higher gust frequencies were increased somewhat. A similar time lag in case 6, however, caused a considerable increase in the amplitude of acceleration due to gusts at frequencies near the natural frequency of the airplane. This undesirable effect results from a reduction in the damping of the oscillations of the airplane due to lag in the control. A discussion of the reasons for these effects will be given in connection with the subject of dynamic stability.

Effect of airspeed.- The effect of variations in airspeed on the behavior of an accelerometer-controlled acceleration alleviator is different from that for a vane-controlled device. In practice, a fixed ratio would exist between the flap deflection and the normal acceleration n, expressed dimensionally. The value of K used in equation (29) relates the flap deflection to the nondimensional acceleration factor $D(\alpha_o - \theta)$. From formula (11),

$$D\left(\alpha_o - \theta\right) = \frac{ngc}{V^2}$$

The value of K is therefore

$$K = \frac{\delta_f}{n}\frac{V^2}{gc}$$

For a given ratio of flap deflection to normal acceleration $\frac{\delta_f}{n}$, the value of K would vary as V^2. If the value of K were selected for the cruising condition, therefore, a smaller value would exist in low-speed flight. Calculations for case 7 made with smaller values of K have shown that the accelerations due to rough air vary approximately inversely with the effective inertia resisting vertical acceleration. A smaller percentage reduction of airplane acceleration would therefore occur at lower airspeed.

Effects of time lag in an accelerometer control cause variations in the characteristics with airspeed, as discussed previously for the vane control. Since the lag expressed in chords would vary directly with the airspeed, the optimum condition of zero lag would be more closely approached at the lower airspeeds. The increase in the lag at higher airspeeds would be expected to cause dynamic instability to set in at some value of airspeed in cases, such as case 6, where lag reduces the dynamic stability. This instability would not be expected, however, in cases such as case 8 where reasonable values of lag do not reduce the dynamic stability. In such cases, the effect of the lag would be to make the acceleration alleviator increasingly less effective in counteracting gusts of short wave length as the airspeed increased.

STATIC AND DYNAMIC STABILITY AND CONTROL CHARACTERISTICS OF AIRPLANES WITH ACCELERATION ALLEVIATORS

Provision of Controllability

Inasmuch as the acceleration alleviators discussed previously tend to reduce or eliminate the lift increment due to change of angle of attack, these devices would cause a conventional elevator control to become ineffective in producing a change in the direction of the flight path. This problem may be overcome by linking the flaps as well as the elevator to the control stick. When the pilot deflects the control stick to produce a pull-up with such an arrangement, the resulting events may be described as follows: First, the flaps move down, producing lift in the desired direction. Then as the airplane responds, the change in angle of attack or acceleration causes the flaps to move back toward their neutral position. It would appear desirable to retain the usual linkage between the control stick and elevator, because the elevator produces pitching moments to rotate the airplane in the desired direction.

A more detailed analysis of the static and dynamic longitudinal stability and control characteristics with such control systems is now presented. This study involves:

(1) A determination of the basic static longitudinal control parameters, namely, the variation of pilot's control position with lift coefficient over the speed range in straight flight and in steady accelerated flight at constant speed

(2) A consideration of the dynamic stability of the airplane with controls fixed following a disturbance

(3) A study of the controllability in maneuvers by calculating the response to an abrupt control deflection

(4) A determination of the stability of the combination of airplane, acceleration alleviator, and human pilot by assuming that the human pilot deflects the control stick in proportion to the angle of pitch.

The problem of control by an autopilot of an airplane equipped with an acceleration alleviator has not been considered in the present analysis, except insofar as item (4) simulates the action of a simple displacement-type autopilot sensitive to angle of pitch. Many possibilities exist for introducing additional signals into an autopilot for controlling the airplane, and these signals might be combined in various ways to operate the elevator, flaps, and throttle. These possibilities form interesting subjects for further study.

Static Longitudinal Stability

Static longitudinal control characteristics with vane control.- A diagram of the control system contemplated for use with the vane control is shown in figure 10. Under steady conditions, the relation between flap, elevator, and vane positions is given by the following equation:

$$\delta_f = K\delta_v + m\delta_e \tag{33}$$

Inasmuch as the control stick is directly linked to the elevator, the elevator position provides a measure of the control-stick position.

The static control characteristics in straight flight may be determined by the use of equation (33) in conjunction with equations (2). In applying these equations to straight flight all terms involving pitching velocity, pitching acceleration, and rate of change of angle of attack are omitted, but a term is added to the equation of vertical forces to represent the airplane weight. The resulting equations may be solved for δ_f, δ_v, or δ_e as a function of the lift coefficient C_L. (In specifying the signs of the terms $\frac{d\delta_f}{dC_L}$ and $\frac{d\delta_e}{dC_L}$, C_L will be considered positive for upward lift, in accordance with the usual convention in airplane performance work.)

The control characteristics in steady accelerated flight at constant speed may be determined in a similar manner by omitting all terms involving rate of change of angle of attack and pitching acceleration, but retaining the terms involving pitching velocity. In this case, the change in lift coefficient from straight flight is given by the relation

$$\Delta C_L = 2\mu D\theta$$

The variations of the desired quantities with lift coefficient may then be obtained.

The resulting expressions for the static control characteristics are as follows: for straight flight,

$$\frac{d\delta_e}{dC_L} = \frac{1}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

$$\frac{d\delta_f}{dC_L} = \frac{K \frac{C_{m_{\delta_e}}}{C_{m_\alpha}} + m}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

for accelerated flight

$$\frac{d\delta_e}{dC_L} = \frac{1 - \frac{C_{m_q} C_{Z_\alpha}}{4\mu C_{m_\alpha}} + \frac{C_{Z_q}}{4\mu}}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

$$\frac{d\delta_f}{dC_L} = \frac{K l_n}{2\mu} + \frac{K\left(\frac{C_{m_{\delta_e}}}{C_{m_\alpha}} + \frac{C_{Z_q} C_{m_{\delta_e}}}{4\mu C_{m_\alpha}} - \frac{C_{m_q} C_{Z_{\delta_e}}}{4\mu C_{m_\alpha}}\right) + m\left(1 - \frac{C_{m_q} C_{Z_\alpha}}{4\mu C_{m_\alpha}} + \frac{C_{Z_q}}{4\mu}\right)}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

In these expressions, all the derivatives represent the modified values of these parameters with the acceleration alleviator in operation. The values of C_{Z_α}, C_{m_α}, C_{Z_q}, and C_{m_q} are given in equations (27). The values of $C_{Z_{\delta_e}}$ and $C_{m_{\delta_e}}$ are as follows:

$$C_{Z_{\delta_e}} = \left(C_{Z_{\delta_e}}\right)_t + mC_{Z_{\delta_f}} \qquad C_{m_{\delta_e}} = \left(C_{m_{\delta_e}}\right)_t + mC_{m_{\delta_f}} \tag{34}$$

where $C_{Z_{\delta_f}}$ and $C_{m_{\delta_f}}$ are given by formulas (8).

Static longitudinal control characteristics with accelerometer control.- The control system contemplated for use with the accelerometer control is shown in figure 15. Under steady conditions, the relation between flap position, elevator position, and normal acceleration is given by the following equation:

$$\delta_f = KD(\alpha_o - \theta) + m\delta_e \tag{35}$$

By the use of this equation in conjunction with equations (2), the static control characteristics for the accelerometer control may be

derived in the same manner as those for the vane control. The resulting expressions for the static control characteristics are as follows:
for straight flight

$$\frac{d\delta_e}{dC_L} = \frac{1}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

$$\frac{d\delta_f}{dC_L} = \frac{m}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

for accelerated flight

$$\frac{d\delta_e}{dC_L} = \frac{1 - \frac{C_{m_q}{}' C_{Z_\alpha}}{4\mu C_{m_\alpha}} + \frac{C_{Z_q}{}'}{4\mu}}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

$$\frac{d\delta_f}{dC_L} = -\frac{K}{2\mu} + \frac{m\left(1 - \frac{C_{m_q}{}' C_{Z_\alpha}}{4\mu C_{m_\alpha}} + \frac{C_{Z_q}{}'}{4\mu}\right)}{C_{m_{\delta_e}} \frac{C_{Z_\alpha}}{C_{m_\alpha}} - C_{Z_{\delta_e}}}$$

In these expressions, the values of C_{Z_α} and C_{m_α} are those for the basic airplane, inasmuch as the accelerometer control does not change these derivatives. The values of $C_{Z_{\delta_e}}$ and $C_{m_{\delta_e}}$ are given by formulas (34). The values of $C_{m_q}{}'$ and $C_{Z_q}{}'$ are given by formulas (32).

Examples of Static Longitudinal Control Characteristics

Inasmuch as the values of the flap parameters and the gearing constant K are largely dictated by considerations of acceleration alleviation, the gearing constant m (equations (33) or (35)) is the only remaining quantity that can be used to vary the stability and control characteristics. The value of m may be selected so that the flaps return to their neutral position in steady accelerated flight. This method of selecting the value of m appears logical because the elevator control required in steady accelerated flight will then be the same as that for the basic airplane. The values of $\frac{d\delta_e}{dC_L}$ and $\frac{d\delta_f}{dC_L}$ in straight flight have been computed for most of the cases considered previously, by using values of m determined in this way. Note that the value of m varies with center-of-gravity position, a condition not likely to be realized in practice. The following tables list values of the static-control parameters for the basic airplane for three center-of-gravity positions and values of the static-control parameters for the airplane with the various acceleration alleviators, with the same center-of-gravity positions:

For the basic airplane:

Static margin, percent M.A.C.	0	10	20
$\frac{d\delta_e}{dC_L}$, straight flight, radians	0	-0.087	-0.178
$\frac{d\delta_e}{dC_L}$, accelerated flight, radians	-0.128	-0.214	-0.306

For the airplane with vane-controlled acceleration alleviator $\left(\frac{d\delta_e}{dC_L}\right.$ in accelerated flight is same as for basic airplane; $\frac{d\delta_f}{dC_L}$ in accelerated flight is $\left.0\right)$:

Case	$\frac{d\delta_e}{dC_L}$, straight flight (radians) for static margin, basic airplane, of -			$\frac{d\delta_f}{dC_L}$, straight flight (radians) for static margin, basic airplane, of -		
	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
2	-0.129	-0.215	-0.305	-0.710	-0.719	-0.723
3	-.130	-.218	-.311	1.199	1.182	1.170
4	Indeterminate			∞	∞	∞
5	-.129	-.209	-.299	-2.835	-3.161	-3.468

For the airplane with accelerometer-controlled acceleration alleviator $\left(\right.$at an airspeed of 200 mph, $\frac{d\delta_e}{dC_L}$ in accelerated flight is same as for basic airplane; $\frac{d\delta_f}{dC_L}$ in accelerated flight is $\left.0\right)$:

Case	$\frac{d\delta_e}{dC_L}$, straight flight (radians) for static margin, basic airplane, of -			$\frac{d\delta_f}{dC_L}$, straight flight (radians) for static margin, basic airplane, of -		
	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
6	0	-0.023	-0.062	0	0.359	0.666
7	0	-.087	-.166	0	1.465	1.946
8	0	-.049	-.144	0	.816	1.338

Vane-controlled acceleration alleviators.- The results given for the vane control are independent of airspeed. The values of $\frac{d\delta_e}{dC_L}$ in straight flight with the vane control are seen to be nearly (though not exactly) equal to the values of $\frac{d\delta_e}{dC_L}$ in accelerated flight for the basic airplane. This condition may be shown to exist whenever the value of m for the vane control is selected to give the same control characteristics in steady accelerated flight as the basic airplane. Because the value of $\frac{d\delta_e}{dC_L}$ in accelerated flight for the basic airplane is of the correct order of magnitude, this condition is desirable.

In most cases which were shown to be favorable for acceleration alleviation, however, the values of $\frac{d\delta_f}{dC_L}$ in straight flight are excessive. In these cases, incorporation of a device to adjust the trim position of the flaps to maintain small flap deflections in straight flight would be necessary, and the values of $\frac{d\delta_e}{dC_L}$ in straight flight would revert to those of the basic airplane.

Accelerometer-controlled acceleration alleviators.- The values of $\frac{d\delta_e}{dC_L}$ and $\frac{d\delta_f}{dC_L}$ in straight flight for the accelerometer control are independent of airspeed with a given value of m. A constant value of m, however, will provide $\frac{d\delta_f}{dC_L}$ equal to zero in accelerated flight only at the selected airspeed of 200 miles per hour. At lower airspeeds, $\frac{d\delta_f}{dC_L}$ would be positive and at higher airspeeds, $\frac{d\delta_f}{dC_L}$ would be negative. The values of $\frac{d\delta_e}{dC_L}$ and $\frac{d\delta_f}{dC_L}$ obtained with the accelerometer control are quite different from those obtained with the vane control. The value of $\frac{d\delta_e}{dC_L}$ in straight flight with zero static margin is zero, as it is in the case of the basic airplane, because there is no change in acceleration or elevator deflection to cause the flaps to move.

Effect of maintaining a constant value of the gearing parameter m.- In the preceding cases, the value of m was changed for each center-of-gravity position to provide a value of $\frac{d\delta_f}{dC_L}$ equal to zero in accelerated flight. In practice, the value of m would be independent of center-of-gravity position. Some additional calculations were therefore made in which the value of m was made constant at the value used with the intermediate center-of-gravity position. The calculations are summarized in the following tables:

For the airplane with vane-controlled acceleration alleviator:

Case	$\frac{d\delta_e}{dC_L}$ (radians) for static margin, basic airplane, of -			$\frac{d\delta_f}{dC_L}$ (radians) for static margin, basic airplane, of -		
	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
5 (accelerated flight)	-0.187	-0.213	-0.246	-1.29	0	1.45
5 (straight flight)	-.189	-.209	-.236	-4.14	-3.16	-1.94

For the airplane with accelerometer-controlled acceleration alleviator:

Case	$\frac{d\delta_e}{dC_L}$ (radians) for static margin, basic airplane, of -			$\frac{d\delta_f}{dC_L}$ (radians) for static margin, basic airplane, of -		
	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.	0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
8 (accelerated flight)	-0.165	-0.214	-0.265	-0.830	0	0.847
8 (straight flight)	0	-.049	-.099	0	.816	1.653

In general, use of a constant value of m does not cause undesirable changes in the value of $\frac{d\delta_e}{dC_L}$. A rather large variation with center-of-gravity position of the value of $\frac{d\delta_f}{dC_L}$ in maneuvers occurs with either the vane or the accelerometer control. These results indicate that the center-of-gravity range with an acceleration alleviator in operation would have to be limited to prevent excessive flap deflections from occurring in steady maneuvers.

Examples of Dynamic Longitudinal Stability Characteristics

The characteristic equation which determines the dynamic longitudinal stability of an airplane with either the vane or accelerometer control is equivalent to the expression obtained by equating to zero the denominator of the equations for the response to sinusoidal gusts. For the vane control, the equations for response are given as equations (25) and (26), and the values of the stability derivatives are given as equations (27). For the accelerometer control, equations for response are given as equations (30) and (31) and the values of the stability derivatives are given as equations (8) and (32). The roots of the characteristic equation determine the period and damping of the various modes of motion. The following table gives the dynamic stability characteristics with control stick fixed for the cases discussed previously:

For the basic airplane:

Static margin, percent M.A.C.	0	10	20
P, sec	-----	6.59	2.76
$T_{1/2}$, sec	$\begin{cases} 0.127 \\ .506 \end{cases}$	0.203	0.203

For the airplane with vane-controlled acceleration alleviator:

Case	Characteristic of motion	Static margin, basic airplane, of -		
		0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
2	P, sec $T_{1/2}$, sec	3.27 0.614	3.30 0.489	3.28 0.429
3	T_2, sec $T_{1/2}$, sec	0.757 0.154	0.753 0.132	0.742 0.118
4	T_2, sec $T_{1/2}$, sec	∞ 0.150	∞ 0.131	∞ 0.117
5	$T_{1/2}$, sec $T_{1/2}$, sec	1.37 0.242	1.63 0.202	1.83 0.169

For the airplane with accelerometer-controlled acceleration alleviator:

Case	Characteristic of motion	Static margin, basic airplane, of -		
		0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
6	P, sec $T_{1/2}$, sec	2.75 0.416	2.63 0.423	2.43 0.431
7	$T_{1/2}$, sec $T_{1/2}$, sec	3.91 0.135	1.82 0.143	1.12 0.153
8	$T_{1/2}$, sec $T_{1/2}$, sec	0.925 0.189	0.633 0.214	0.414 0.268

Basic airplane.- The motion of the basic airplane with zero static margin following a disturbance is a rapid convergence. This motion becomes a very heavily damped oscillation as the static stability is increased.

Vane-controlled acceleration alleviators.- The examples chosen illustrate the wide variations in dynamic stability characteristics that may be obtained with vane-controlled acceleration alleviators. In case 2, the use of a positive value of $\frac{\partial \epsilon}{\partial \delta_f}$ has resulted in a negative, or stabilizing, increment of C_{m_α} and has therefore reduced the period of the oscillation below that of the basic airplane. This restoring moment lags behind the angle of attack, however, as a result of lag in downwash, and thereby reduces the damping. Increasing the value of nondimensional time lag τ (as would occur with increasing airspeed) tends to reduce further the damping under these conditions, and unstable oscillations might occur at sufficiently high values of airspeed.

In case 3, efforts to reduce the response of the airplane to gusts by use of a negative value of $\left(C_{m_{\delta_f}}\right)_w$ in conjunction with a value of $\frac{\partial \epsilon}{\partial \delta_f}$ of zero have resulted in a positive, or unstable, value of C_{m_α}. The airplane performs a rapid divergence following a disturbance. With the value of K used, the value of C_{Z_α} is slightly positive (opposite to the normal case). The divergence would therefore consist, for example, of a nosing-up together with a loss of altitude. The static control characteristics for this case were normal. This example illustrates that the variations of elevator angle with lift coefficient in straight and accelerated flight, while they are important as control characteristics, cannot be interpreted as stability characteristics for an airplane acceleration alleviator. This conclusion is in contrast to the case for a normal airplane, because the values of $\frac{d\delta_e}{dC_L}$ in straight and accelerated flight are normally measured in flight to determine the static stability. The effect of this unusual combination of stability and control characteristics on the ability of a pilot to control the airplane is considered subsequently.

In case 4, which is optimum for acceleration alleviation, the values of C_{Z_α} and C_{m_α} have been reduced to zero. These characteristics were obtained by use of a positive value of $\left(C_{m_{\delta_f}}\right)_w$ in conjunction with a negative value of $\frac{\partial \epsilon}{\partial \delta_f}$. The airplane is neutrally stable in one mode and, therefore, has no tendency to return to the initial angle of attack following a disturbance. The presence of a second mode of rapid convergence, however, indicates that any pitching velocity imparted to the airplane will be quickly damped out.

The use of values of C_{Z_α} and C_{m_α} of zero appears undesirable, because the pilot would be entirely dependent on external or instrument references to adjust the attitude angle of the fuselage in flight at a given speed and continual attention would be required to keep this angle from changing. By use of a small negative value of C_{m_α}, this difficulty might be avoided without much sacrifice in acceleration alleviation. (In practice, the sacrifice in acceleration alleviation might not be apparent, because nonuniformity of gusts across the span would prevent even the optimum theoretical arrangement from reducing the accelerations exactly to zero.) For this reason, the flap characteristics of case 5 were selected to give a small negative value of C_{m_α}. This condition was obtained by use of a small negative value of $\left(C_{m_{\delta_f}}\right)_w$, in conjunction with a value of $\frac{\partial\epsilon}{\partial\delta_f}$ of zero. The motion following a disturbance for this case is a rapid convergence. The response to gusts for this arrangement has been discussed previously and was shown to be favorable. In addition, the static control characteristics were shown to be satisfactory, with the possible exception that rather large negative values of $\frac{d\delta_f}{dC_L}$ existed in straight flight.

Accelerometer-controlled acceleration alleviators.- Cases 6, 7, and 8 for the accelerometer control have the same flap characteristics as cases 2, 4, and 5, respectively, for the vane control. Similarity with the results for the vane control will be noted in that cases 2 and 6 show less damping than the basic airplane and cases 5 and 8 exhibit rapid convergences. The main reason for any differences between the results for the vane and accelerometer controls is the lack of effect of the accelerometer control on the value of C_{m_α}. As a result, the conditions which produce zero or positive values of C_{m_α} and, hence, neutral or straight diverging motions in the case of the vane control produce convergences in the case of the accelerometer control. (Compare cases 4 and 7.)

Effect of time lag in accelerometer-controlled acceleration alleviators.- The effect of a time lag in the booster on the dynamic stability with an accelerometer control has been investigated. The lag was taken into account by the approximate method discussed previously. The method may be subject to error in this case because the lag in the booster is added to the lag in downwash in determining moment variations at the tail due to downwash from the flaps. The resulting over-all lag is somewhat larger than can be accurately taken into account by the approximate method. The results, therefore, may not be quantitatively correct for the shorter-period modes of motion, but the trends are believed to be shown correctly. The effects of time lag in the booster for accelerometer-controlled acceleration alleviators are given in the following table:

Case	τ (chords)	Characteristic of motion	Static margin, basic airplane, of -		
			0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
6	0	P, sec $T_{1/2}$, sec	2.75 0.416	2.63 0.423	2.43 0.431
	2	P, sec $T_{1/2}$, sec	1.94 0.613	1.90 0.647	1.85 0.674
	4	P, sec $T_{1/2}$, sec	2.08 3.51	2.02 3.58	1.96 3.33
8	0	$T_{1/2}$, sec $T_{1/2}$, sec	0.925 0.189	0.633 0.214	0.414 0.268
	2	$T_{1/2}$, sec $T_{1/2}$, sec	0.924 0.189	0.648 0.213	0.442 0.259
	4	$T_{1/2}$, sec $T_{1/2}$, sec	0.930 0.186	0.186 0.214	0.460 0.262

At an airspeed of 200 miles per hour, a value of τ of 2 chords represents a time lag of 0.061 second and a value of τ of 4 chords represents a time lag of 0.122 second. The results show that the longitudinal oscillation in case 6 becomes more poorly damped with increasing time lag and would probably become unstable with further increase in lag. On the other hand, increasing time lag in case 8 has a negligible effect on the damping of the rapid convergence. The effect of a small time lag in a control operating the flaps to oppose changes in normal acceleration is not likely to cause instability of vertical motions of the airplane directly because any lag in the flap operation will create a force opposing the vertical velocity, which will tend to damp the oscillations. An unstable pitching oscillation may be obtained, however, if the over-all pitching moment due to flap deflection is positive. In this case, the flap deflection in response to changes in angle of pitch causes a moment tending to oppose the pitching displacement of the airplane. Any lag in this moment will create a moment in the same direction as the pitching velocity, which will tend to build up the oscillations. In case 6 this condition exists; whereas, in case 8, in which the pitching moment due to flap deflection has a small or negative value over the center-of-gravity range investigated, this instability is avoided.

Because a time lag would exist in any practical booster arrangement, the use of flap characteristics similar to those of case 8 would be necessary with an accelerometer control. These flap characteristics have been shown previously to be desirable for acceleration alleviation.

Response to Control Deflection

The response of the airplane to a control deflection may be determined by solving the equations of motion for the normal acceleration or pitching velocity in terms of the elevator angle. When an acceleration alleviator is used, the applicable relation governing the variation of flap angle with elevator angle must be substituted in the equations of motion.

Response to control deflection with vane control.- In the control system contemplated for use with the vane control (fig. 10) the flaps are linked to the elevator through a booster. Any time lag in the booster will therefore result in the flap motion lagging behind the elevator motion. The relation between flap, elevator, and vane positions is given by the following equation:

$$\delta_f = \left(K\delta_v + m\delta_e\right)e^{-\tau D}$$

If this equation is substituted into equations (2) by the methods discussed previously, the resulting equations can be solved for the normal acceleration or the pitching velocity in terms of the elevator angle $\left(\alpha_g\right.$ is set equal to zero$\left.\right)$. The transfer functions for $D\theta$ and and n in terms of δ_e are as follows:

$$\frac{n}{\delta_e} = \frac{1}{F}\frac{\Delta_{13}}{\Delta_7}$$

$$\frac{D\theta}{\delta_e} = \frac{\Delta_{14}}{\Delta_7}$$

where

$$\Delta_{13} = D^3\left(-\mu K_y{}^2 C_{Z_{D\delta_e}} + \frac{1}{8} C_{Z_{D\delta_e}} C_{m_{Dq}} - \frac{1}{8} C_{m_{D\delta_e}} C_{Z_{Dq}}\right) + D^2\left[-2\mu K_y{}^2 C_{Z_{\delta_e}} + \right.$$

$$\left.\frac{1}{4} C_{Z_{\delta_e}} C_{m_{Dq}} - \frac{1}{4} C_{m_{\delta_e}} C_{Z_{Dq}} + \frac{1}{4} C_{Z_{D\delta_e}}\left(C_{m_q} + C_{m_{D\alpha}}\right) - \frac{1}{4} C_{m_{D\delta_e}}\left(C_{Z_q} + C_{Z_{D\alpha}}\right)\right] +$$

$$D\left[\frac{1}{2} C_{Z_{\delta_e}}\left(C_{m_q} + C_{m_{D\alpha}}\right) - \frac{1}{2} C_{m_{\delta_e}}\left(C_{Z_q} + C_{Z_{D\alpha}}\right) + \frac{1}{2} C_{Z_{D\delta_e}} C_{m_\alpha} - \frac{1}{2} C_{m_{D\delta_e}} C_{Z_\alpha}\right] +$$

$$C_{m_\alpha} C_{Z_{\delta_e}} - C_{Z_\alpha} C_{m_{\delta_e}}$$

$$\Delta_{14} = D^2\left(-\mu C_{m_{D\delta_e}} + \frac{1}{4} C_{m_{D\delta_e}} C_{Z_{D\alpha}} - \frac{1}{4} C_{Z_{D\delta_e}} C_{m_{D\alpha}}\right) +$$

$$D\left(-2\mu C_{m_{\delta_e}} + \frac{1}{2} C_{Z_{D\alpha}} C_{m_{\delta_e}} - \frac{1}{2} C_{m_{D\alpha}} C_{Z_{\delta_e}} + \frac{1}{2} C_{m_{D\delta_e}} C_{Z_\alpha} - \frac{1}{2} C_{Z_{D\delta_e}} C_{m_\alpha}\right) +$$

$$C_{Z_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{Z_{\delta_e}}$$

The stability derivatives are listed in equations (27) and (34). The values of two previously undefined derivatives are as follows:

$$C_{Z_{D\delta_e}} = 2m \frac{\partial \epsilon}{\partial \delta_f} C_{Z_{\alpha_t}} (\tau + l) - 2m\tau \left(C_{Z_{\delta_f}}\right)_w$$

$$C_{m_{D\delta_e}} = 2m \frac{\partial \epsilon}{\partial \delta_f} C_{m_{\alpha_t}} (\tau + l) - 2m\tau \left(C_{m_{\delta_f}}\right)_w$$

Response to control deflection with accelerometer control.- In the case of the accelerometer control (fig. 15), the flaps are again linked to the elevator through a booster. In the response calculations made previously, however, lag in the booster was neglected. In this case the relation between flap position, elevator position, and normal acceleration is given by equation (35). The transfer functions for $D\theta$ and n in terms of δ_e are as follows:

$$\frac{n}{\delta_e} = \frac{1}{F}\frac{\Delta_{15}}{\Delta_{10}}$$

$$\frac{D\theta}{\delta_e} = \frac{\Delta_{16}}{\Delta_{10}}$$

where

$$\Delta_{15} = D^3\left(-\mu K_y{}^2 C_{Z_{D\delta_e}}\right) + D^2\left(-2\mu K_y{}^2 C_{Z_{\delta_e}}\right) +$$

$$D\left[\frac{1}{2}\, C_{Z_{\delta_e}}\left(C_{m_q} + C_{m_{D\alpha}}\right) - \frac{1}{2}\, C_{m_{\delta_e}}\left(C_{Z_q} + C_{Z_{D\alpha}}\right) + \frac{1}{2}\, C_{Z_{D\delta_e}} C_{m_\alpha} - \frac{1}{2}\, C_{m_{D\delta_e}} C_{Z_\alpha}\right] +$$

$$C_{m_\alpha} C_{Z_{\delta_e}} - C_{Z_\alpha} C_{m_{\delta_e}}$$

$$\Delta_{16} = D^2\left(-\mu C_{m_{D\delta_e}} + \frac{1}{4}\, C_{Z_{D^2\alpha}}{}' C_{m_{\delta_e}} - \frac{1}{4}\, C_{m_{D^2\alpha}}{}' C_{Z_{\delta_e}} + \frac{1}{4}\, C_{m_{D\delta_e}} C_{Z_{D\alpha}}{}' - \frac{1}{4}\, C_{Z_{D\delta_e}} C_{m_{D\alpha}}{}'\right) +$$

$$D\left(-2\mu C_{m_{\delta_e}} + \frac{1}{2}\, C_{Z_{D\alpha}}{}' C_{m_{\delta_e}} - \frac{1}{2}\, C_{m_{D\alpha}}{}' C_{Z_{\delta_e}} + \frac{1}{2}\, C_{m_{D\delta_e}} C_{Z_\alpha} - \frac{1}{2}\, C_{Z_{D\delta_e}} C_{m_\alpha}\right) +$$

$$C_{Z_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{Z_{\delta_e}}$$

The stability derivatives are listed in equations (8), (32), and (34). The values of $C_{Z_{D\delta_e}}$ and $C_{m_{D\delta_e}}$ are as follows:

$$C_{Z_{D\delta_e}} = 2ml \frac{\partial \epsilon}{\partial \delta_f} C_{Z_{\alpha_t}}$$

$$C_{m_{D\delta_e}} = 2ml \frac{\partial \epsilon}{\partial \delta_f} C_{m_{\alpha_t}}$$

Examples of Response to Control Deflection

From the transfer functions, the steady-state response of the airplane to sinusoidal control motions may be determined as described previously or the transient response to a step-function control motion may be determined by the Laplace transform method (reference 22). The response to step-function control motions has been selected to illustrate the various cases because this type of control motion is more representative of the control motions normally used in maneuvering.

Basic airplane.- The response of the basic airplane to a step function of the elevator is shown in figure 19. The response is heavily damped, in accordance with the dynamic stability characteristics presented previously, and shows an appreciable lag between deflection of the control and the attainment of a steady value of normal acceleration. This lag results from the necessity of pitching the airplane to a new angle of attack in order to develop increased lift. The slight negative acceleration at the start of the maneuver is caused by the downward lift on the tail following the abrupt elevator deflection.

Vane-controlled acceleration alleviator.- The response of the airplane with the vane-controlled acceleration alleviator of case 5 to a step-function motion of the elevator is shown in figure 20. The development of normal acceleration is much more rapid than for the basic airplane because of the production of lift immediately upon deflection of the flaps. Nevertheless, the airplane has no tendency to overshoot the final acceleration.

The mathematical solution for this case indicates an initial impulsive negative lift and an impulsive moment applied to the airplane because of the terms $C_{Z_{D\delta_e}}$ and $C_{m_{D\delta_e}}$ and the infinite value of $D\delta_e$

at the start of the step motion. The impulsive pitching moment results in a small finite value of angular velocity at the start of the motion. Predictions of the motion directly after a discontinuous motion of the elevator are not given accurately by the foregoing theory, however, because such motion involves high-frequency portions of the response spectrum where the approximations for time lag do not apply. In practice, the lift would follow the control motion in the correct direction (except for the effect of lift on the tail due to the elevator deflection) but with a slight lag resulting from lag in the booster. This variation is indicated approximately by the dotted lines in figure 20. Additional lag would be introduced by unsteady-lift effects.

The effects of reducing the static stability on the basic airplane are to increase the acceleration and the time required for the airplane to reach a steady acceleration following a deflection of the elevator (fig. 19). The airplane with the vane-controlled acceleration alleviator of case 5 reaches a desirable value of steady acceleration quickly in spite of the fact that the value of $C_{m_{\alpha}}$ is very low, corresponding to a value which would give a static margin of only 3.6 percent of the mean aerodynamic chord of the basic airplane. This result indicates that the use of a low value of $C_{m_{\alpha}}$, which was shown to be desirable with the vane control for acceleration alleviation, would not adversely affect the control characteristics in maneuvers.

Accelerometer-controlled acceleration alleviator.- The response of the airplane with the accelerometer-controlled acceleration alleviator of case 8 to a step-function motion of the elevator is shown in figure 21. The results in this case are very similar to those for the vane control of case 5. No lag is indicated at the start of the motion because no lag was assumed in the booster in this case. In practice, however, a small lag would exist because of lag in the booster and unsteady-lift effects.

Incorporation of g restrictor.- A point of interest somewhat incidental to the present analysis is the ease with which a device to limit the maneuvering load factor could be incorporated in an airplane with an acceleration alleviator. This limitation could be accomplished, for example, by a mechanism which would prevent further rearward movement of the control stick when a given acceleration was reached. Devices of this type, called g restrictors, have proved unsatisfactory in conventional airplanes because of the lag between the control movement and the resulting acceleration. Thus in a conventional airplane, the pilot may move the control stick to a large deflection before the acceleration has increased appreciably. Limitation of the movement of the control stick, or even reversal of the movement, when the limit load factor is reached will not prevent further increase of acceleration because the control stick is already far beyond the position required to develop the limit load factor. With the acceleration alleviator in operation, however, the lag between the control movement and the resulting acceleration is so small that satisfactory operation of such a device should be possible.

Stability of the Airplane Equipped with Acceleration Alleviator under Control of a Pilot

Even when the airplane equipped with an acceleration alleviator is stable with controls fixed, instability might occur when the system is controlled by a human or automatic pilot because of the lag introduced by the booster between the pilot's action and the flap motion. The stability of the controlled airplane therefore requires investigation. The ability of a pilot to stabilize systems which are unstable with controls fixed is also of interest. For these reasons, calculations of the stability of controlled motion for some of the cases discussed previously have been made.

In these calculations, the pilot's control was assumed to be moved in proportion to and opposing the angle of pitch, an action equivalent to that of a displacement autopilot with no lag. This controlling action may also be assumed as a reasonable approximation to that of a human pilot. An elevator movement of 1° per degree angle of pitch was assumed. The linkage between the elevator and flaps was similar to that used previously in calculating the static control characteristics. The calculations were made for the basic airplane and for three cases with the vane control. No calculations were made for the accelerometer control. The following table gives the dynamic stability characteristics of the controlled airplane:

For the basic airplane:

Static margin, percent M.A.C.		0	10	20
Oscillation	P, sec	2.69	2.28	1.77
	$T_{1/2}$, sec	0.539	0.320	0.266
Convergence $T_{1/2}$, sec		0.171	0.301	0.490

For the airplane with vane-controlled acceleration alleviator:

Case	Characteristic of motion	Static margin, basic airplane, of -		
		0 M.A.C.	0.10 M.A.C.	0.20 M.A.C.
2	Oscillation: P, sec	1.56	1.69	1.90
	Oscillation: $T_{1/2}$, sec	22.6	2.29	1.37
	Convergence $T_{1/2}$, sec	0.772	0.690	0.555
3	Oscillation: P, sec	2.39	6.44	(a)
	Oscillation: $T_{1/2}$, sec	0.276	0.236	
	Divergence T_2, sec	0.479	0.503	0.524
5	Oscillation: P, sec	1.88	2.22	2.93
	Oscillation: $T_{1/2}$, sec	0.456	0.419	0.391
	Convergence $T_{1/2}$, sec	2.40	2.08	1.65

[a]Convergences; $T_{1/2}$ = 0.45 and 0.137.

Basic airplane.- The effect of the assumed controlling action on the basic airplane is to reduce the period below that of the uncontrolled airplane and to reduce the damping somewhat. An additional mode of rapid convergence is introduced by the control. The stability of the controlled system is satisfactory because of the rapid damping of both modes of motion.

Vane-controlled acceleration alleviators.- The vane-controlled acceleration alleviator of case 2 was shown to reduce the damping of the uncontrolled motion. The assumed controlling action considerably decreases the period of the oscillation and further reduces the damping, so that for rearward center-of-gravity positions a practically continuous oscillation occurs. This reduction of damping may be attributed to the pitching moments resulting from flap deflection when the flaps move in accordance with the control motion. These moments are in a direction to oppose the changes in angle of pitch and lag behind this angle because of the effects of lag in the booster and lag of downwash. The poor damping of the resulting oscillation would make the airplane difficult to control.

The vane-controlled acceleration alleviator of case 3 was shown to produce a rapid straight divergence with controls fixed. The assumed controlling action introduces a well-damped oscillation but results in an even more unstable mode of straight divergence. This divergence would consist, as before, of a nosing-up together with a loss of altitude because the application of down elevator would cause upward flap deflection resulting in further loss of wing lift.

In a conventional airplane, a control which moves the elevators to oppose the angle of pitch may be used to provide stability even when static instability is present with controls fixed. This stabilizing ability, which depends on the presence of a normal value of C_{Z_α}, does not exist when the value of C_{Z_α} is reduced to zero by means of an acceleration alleviator. An airplane with a vane-controlled acceleration alleviator would therefore probably be uncontrollable by a human pilot with normal reactions unless the value of C_{m_α} were negative.

The vane-controlled acceleration alleviator of case 5 was shown to produce convergent modes of motion with controls fixed. The assumed controlling action causes a well-damped oscillation very similar to that on the basic airplane, and a convergence. This convergence is somewhat slower than that of the basic airplane but appears to be sufficiently rapid to be satisfactory.

CONCLUDING REMARKS

The object of the foregoing study has been the determination of means for reducing the accelerations of an airplane caused by rough air. Inasmuch as any variation of gust velocity with time may be resolved into a series of sinusoidal components with various frequencies and amplitudes, the problem may be considered as that of reducing the response of the airplane to sinusoidal gusts of any frequency. In practice, two types of limitations are encountered on the ability of a mechanism to reduce the response to such gusts. First, there is an upper limit to the frequency of gusts to which the mechanism can correctly respond. If this limitation is not imposed by the mechanism itself, it may be set by the structural frequencies of the airplane for, if a mechanism sensitive to airplane motion operates the controls at frequencies near these structural frequencies, self-excited oscillations similar to flutter are likely to occur. The second limitation concerns the amplitude of gusts that can be handled. Any type of control surface used in conjunction with an alleviating mechanism fails to give increase in lift beyond a certain deflection. A limitation of the deflection range considerably below this value may be required for structural reasons. The lift increment available for offsetting the effects of gusts may therefore be less than that required for handling the most severe gusts encountered. In spite of such limitations, however, it should be possible to offset the effects of gusts completely up to certain limits of amplitude and frequency. Such a result would be expected to improve passenger comfort considerably because small-amplitude gusts are encountered frequently in rough air and because long-period or low-frequency variations in normal acceleration have been shown to be primarily responsible for airsickness.

Consideration of the various means that have been proposed for acceleration alleviation indicates that this objective may be accomplished most simply by operating trailing-edge flaps on the wing by an automatic control mechanism. The characteristics required of such flaps may be seen by considering the sequence of events which occurs when the airplane passes through an upward gust. When the wing reaches the gust, the flaps must be deflected up to produce a lift increment opposing that from the change of angle of attack. In order to avoid undesirable pitching motions of the airplane, it is necessary that the pitching moment produced by the flaps about the wing aerodynamic center should be zero. Then, when the tail penetrates the gust, it is necessary that the downwash due to flap deflection combined with the downwash due to wing lift should just offset the gust velocity at the tail. These characteristics are not obtainable with ordinary flap designs, since such flaps produce a large pitching moment about the wing aerodynamic center and produce downwash in the direction opposite from that required to offset the effect of the gust at the tail. The desired characteristics might be obtained, however, by some modifications to the usual flap design. The flap pitching moment could be reduced to zero by linking the flaps directly to the elevator (or to a portion of the elevator). The reversed direction of downwash due to flap deflection might be obtained by linking a portion of each flap near the fuselage to deflect in the direction opposite from the main portion of the flap further outboard. These modifications would reduce the lift effectiveness of the flaps and would therefore tend to limit the amplitude of gust disturbances which could be completely offset.

Two methods have been considered for sensing the effect of gusts. One of these is the use of a vane mounted on a boom ahead of the nose to detect angle-of-attack changes due to gusts. The other is the use of an accelerometer to detect the accelerations caused by gusts. Both of these methods would require that the small forces produced by the sensing device be amplified by a servomechanism in order to operate the flaps. The use of the vane has the advantage that it provides a small amount of anticipation of the gusts. The lag in the servomechanism could then be adjusted to compensate for the time required for the gust to travel from the vane to the wing.

With the acceleration alleviator in operation, a problem of longitudinal control arises. Normally, control of the airplane by the elevator is accomplished by changing the angle of attack. If the lift increment due to changing the angle of attack is eliminated, the elevator will be ineffective in producing a change in the direction of the flight path. This problem may be overcome by arranging the flaps to deflect in accordance with control-stick deflections as well as with the indications of the gust-sensing device. When the pilot deflects the elevator to make a pull-up with such an arrangement, the resulting events may be described as follows: First, the flaps move down, producing lift in the desired direction; then, as the airplane rotates in response to the elevator, the angle of attack and acceleration increase and cause the flaps to move back to their neutral position.

Calculations have been made of the response to gusts, the response to control deflection, and the static and dynamic stability characteristics of an airplane with an acceleration alleviator in operation and with various assumptions as to the characteristics of the mechanism and aerodynamic parameters of the flaps. In general, it is found that flaps with conventional characteristics are unsatisfactory as acceleration alleviators, because they produce large pitching motions of the airplane in flight through gusts. In addition, the dynamic stability characteristics of the system are unsatisfactory. With flaps modified to reduce their pitching moments and to eliminate or reverse the downwash due to flap deflection, however, excellent acceleration alleviation may be obtained with either the vane or accelerometer sensing device. Satisfactory stability and control characteristics may also be obtained by suitable choice of the flap and control-system parameters.

The response to elevator control of an airplane equipped with an acceleration alleviator is much more rapid than that of a conventional airplane because lift is produced immediately upon deflection of the control stick. The reduction of lag in response to control deflection offers the possibility of incorporating a simple type of device to limit the maneuvering loads on the airplane.

The foregoing theoretical study indicates that consideration must be given to problems of stability and control in the design of acceleration alleviation systems, and that these conditions place rather strict limits on the aerodynamic and control-system parameters that may be successfully employed. The design of an actual system is expected to introduce additional problems of structural and mechanical design that are difficult to foresee. Nevertheless, the promising results obtained in the theoretical investigation indicate that experimental research to verify these results would be desirable.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., March 28, 1951

REFERENCES

1. McFarland, Ross A.: Human Factors in Air Transport Design. McGraw-Hill Book Co., Inc., 1946, pp. 367-377.

2. Alexander, S. J., Cotzin, M., Hill, C. J., Jr., Ricciuti, E. A., and Wendt, G. R.: Wesleyan Universtiy Studies of Motion Sickness.

   I. The Effects of Variation of Time Intervals between Acclerations upon Sickness Rates. Jour. Psychology, vol. 19, Jan. 1945, pp. 49-62.

   II. A Second Approach to the Problem of the Effects of Variation of Time Intervals between Accelerations upon Sickness Rates. Jour. Psychology, vol. 19, Jan. 1945, pp. 63-68.

   III. The Effects of Various Accelerations upon Sickness Rates. Jour. Psychology, vol. 20, July 1945, pp. 3-8.

   IV. The Effects of Waves Containing Two Acceleration Levels upon Sickness. Jour. Psychology, vol. 20, 1945, pp. 9-18.

3. Shufflebarger, C. C.: Tests of a Gust-Alleviating Wing in the Gust Tunnel. NACA TN 802, 1941.

4. Reisert, Thomas D.: Gust-Tunnel Investigation of a Flexible-Wing Model with Semichord Line Swept Back 45°. NACA TN 1959, 1949.

5. Neville, Leslie E.: Designs at the National Air Races. Aviation, vol. 29, no. 4, Oct. 1930, p. 211.

6. Hirsch, René: Sur la stabilisation automatique des avions. Comptes Rendus, t. 204, no. 24, June 14, 1937, pp. 1794-1796.

7. Hirsch, René: Stabilisation des avions. Comptes Rendus, t. 206, no. 3, Jan. 17, 1938, pp. 161-164.

8. Donely, Philip, and Shufflebarger, C. C.: Tests of a Gust-Alleviating Flap in the Gust Tunnel. NACA TN 745, 1940.

9. Russell, A. E.: Some Engineering Problems of Large Aircraft. Jour. R.A.S., vol. 51, no. 434, Feb. 1947, pp. 145-170.

10. Mickleboro, Harry C.: Evaluation of a Fixed Spoiler as a Gust Alleviator. NACA TN 1753, 1948.

11. Kraft, Christopher C., Jr., and Assadourian, Arthur: Experimental Study of an Angle-of-Attack Vane Mounted Ahead of the Nose of an Airplane for Use as a Sensing Device for an Acceleration Alleviator. NACA TN 2415, 1951.

12. Wilson, Edwin Bidwell: Theory of an Aeroplane Encountering Gusts.

    I. NACA Rep. 1, pt. 2, 1915.

    II. NACA Rep. 21, 1917.

    III. NACA Rep. 27, 1918.

13. Bryan, G. H.: Stability in Aviation. Macmillan and Co., Ltd., 1911.

14. Bairstow, L., Jones, B. Melvill, and Thompson, A. W. H.: Investigation into the Stability of an Aeroplane, with an Examination into the Conditions Necessary in Order That the Symmetric and Asymmetric Oscillations can be Considered Independently. R. & M. No. 77, British A.C.A., 1913.

15. Küssner, Hans Georg: Stresses Produced in Airplane Wings by Gusts. NACA TM 654, 1932.

16. Jones, Robert T., and Cohen, Doris: An Analysis of the Stability of an Airplane with Free Controls. NACA Rep. 709, 1941.

17. Breuhaus, W. O.: Effect of Speed Changes on the Normal Acceleration of an Airplane during Sinusoidal Longitudinal Oscillations at Frequencies Ranging from Zero to Eight Radians per Second. FRM No. 35, Flight Res. Dept., Cornell Aero. Lab., Dec. 8, 1947.

18. Cowley, W. L., and Glauert, H.: The Effect of the Lag of the Downwash on the Longitudinal Stability of an Aeroplane and on the Rotary Derivative M_q. R. & M. No. 718, British A.R.C., 1921.

19. Imlay, Frederick H.: A Theoretical Study of Lateral Stability with an Automatic Pilot. NACA Rep. 693, 1940.

20. Lin, Shih-Nge.: A Mathematical Study of the Controlled Motion of Airplanes. M.I.T. Thesis, vol. I, 1939.

21. Jones, Robert T., and Fehlner, Leo F.: Transient Effects of the Wing Wake on the Horizontal Tail. NACA TN 771, 1940.

22. Gardner, Murray F., and Barnes, John L.: Transients in Linear Systems Studied by the Laplace Transformation. Vol. I, John Wiley & Sons, Inc., 1942.

23. Moskovitz, A. I.: XC-35 Gust Research Project - Preliminary Analysis of the Lateral Distribution of Gust Velocity along the Span of an Airplane. NACA RB, March 1943.

24. Jones, Robert T.: A Simplified Application of the Method of Operators to the Calculation of Disturbed Motions of an Airplane. NACA Rep. 560, 1936.

TABLE I

CHARACTERISTICS OF AIRPLANE USED IN CALCULATIONS

Dimensional data:	
Weight, pounds	22,000
Wing area, S, square feet	990
Tail area, S_t, square feet	180
Wing chord, c, feet	9
Tail length, feet	37
Distance from center of gravity to angle-of-attack vane, feet	20
Radius of gyration about Y-axis, k_y, feet	12.33
Nondimensional parameters:	
l	4.12
l_n	2.22
μ (at sea level)	32.2
K_y	1.37
Neutral-point location, percent M.A.C.	40
Aerodynamic parameters of components:	
$C_{Z_{\alpha_w}}$ per radian	-5.21
$C_{Z_{\alpha_t}}$ per radian	-0.571
$C_{m_{\alpha_t}}$ per radian	-2.36
$C_{Z_{\delta_{e_t}}}$ per radian	-0.286
$C_{m_{\delta_{e_t}}}$ per radian	-1.18
$\frac{\partial \epsilon}{\partial \alpha}$	0.5
Aerodynamic center of wing-fuselage combination, percent M.A.C.	18.2

TABLE II

VALUES OF STABILITY DERIVATIVES FOR BASIC AIRPLANE

Quantities dependent on center-of-gravity position:			
Center-of-gravity position, percent M.A.C.	40	30	20
Static margin, percent M.A.C.	0	10	20
C_{m_α}	0	-0.55	-1.10

Quantities independent of center-of-gravity position:	
$C_{m_{D\alpha}}$	-9.70
C_{m_q}	-19.40
$C_{m_{\delta_e}}$	-1.18
C_{Z_α}	-5.5
$C_{Z_{D\alpha}}$	-2.36
C_{Z_q}	-4.72
$C_{Z_{\delta_e}}$	-0.286

TABLE III

FLAP CHARACTERISTICS AND AUTOMATIC-CONTROL CHARACTERISTICS FOR THE VANE-CONTROLLED AND ACCELEROMETER-CONTROLLED ACCELERATION ALLEVIATORS USED IN CALCULATIONS AND FIGURES SHOWING THE RESPONSE TO GUSTS FOR EACH CASE

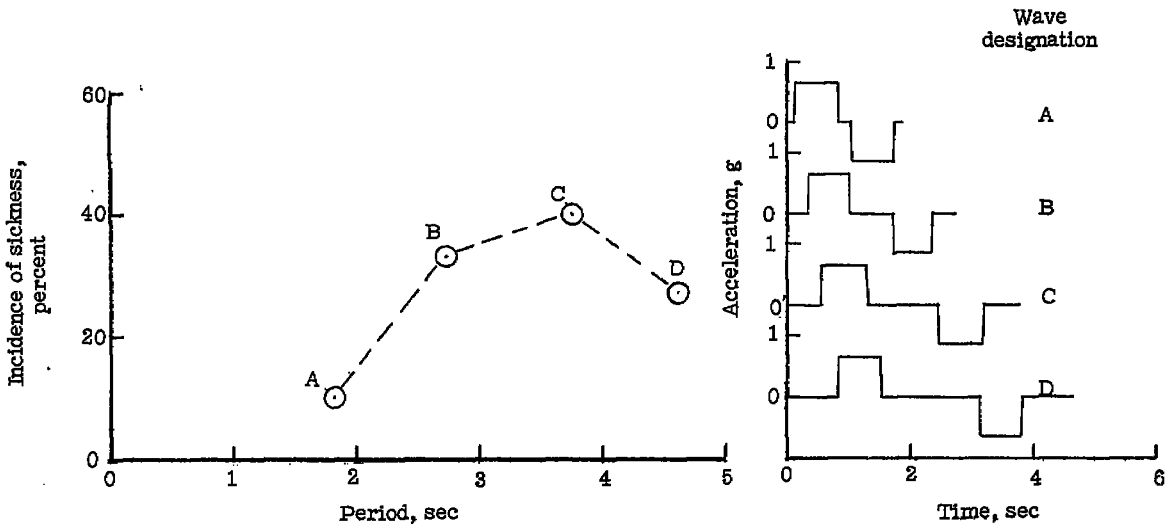
(a) Vane-Controlled Acceleration Alleviators

Case	$\left(C_{Z_{\delta_f}}\right)_w$	$\left(C_{m_{\delta_f}}\right)_w$	$\frac{\partial \epsilon}{\partial \delta_f}$	τ	K	Figure
1	-1.40	-0.338	0.271	2.22	2.74	11
2	-1.40	-.338	.144	2.22	3.37	12
3	-1.40	-.338	0	2.05	4.51	13
4	-1.40	0	-.134	2.22	3.65	--
5	-1.40	-.156	0	2.22	3.72	14

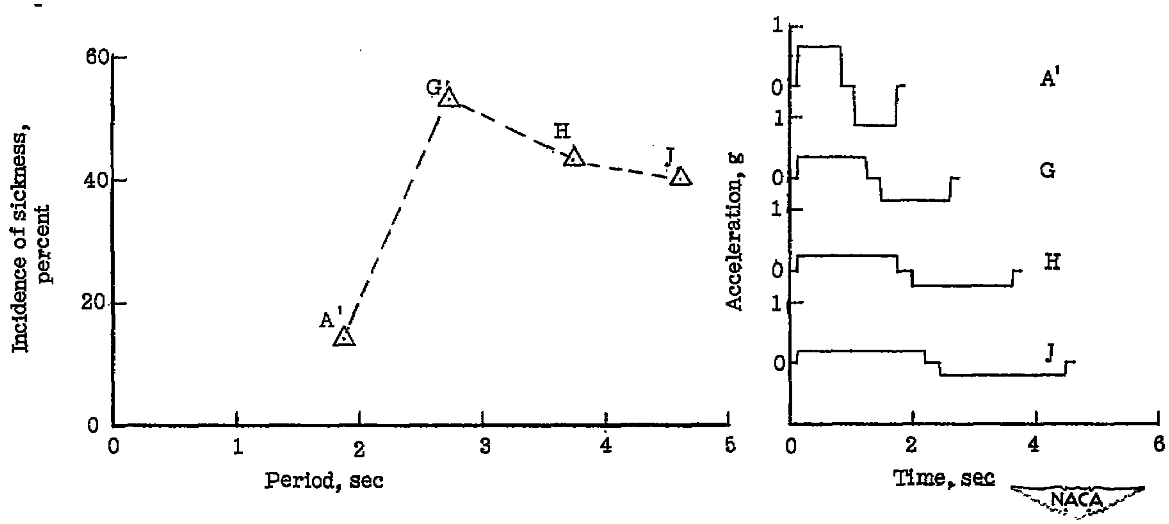
(b) Accelerometer-Controlled Acceleration Alleviators

Case	$\left(C_{Z_{\delta_f}}\right)_w$	$\left(C_{m_{\delta_f}}\right)_w$	$\frac{\partial \epsilon}{\partial \delta_f}$	K	Figure
6	-1.40	-0.338	0.144	208	16
7	-1.40	0	-.134	231	17
8	-1.40	-.156	0	231	18

NACA



(a) Varying interval of zero acceleration.



(b) Varying value of acceleration.

Figure 1.- Percentage of men who became sick within a period of 20 minutes when subjected to vertical oscillations of various periods and wave forms. Various symbols indicate different series of tests. Primes designate repeat tests. Results obtained from reference 2.

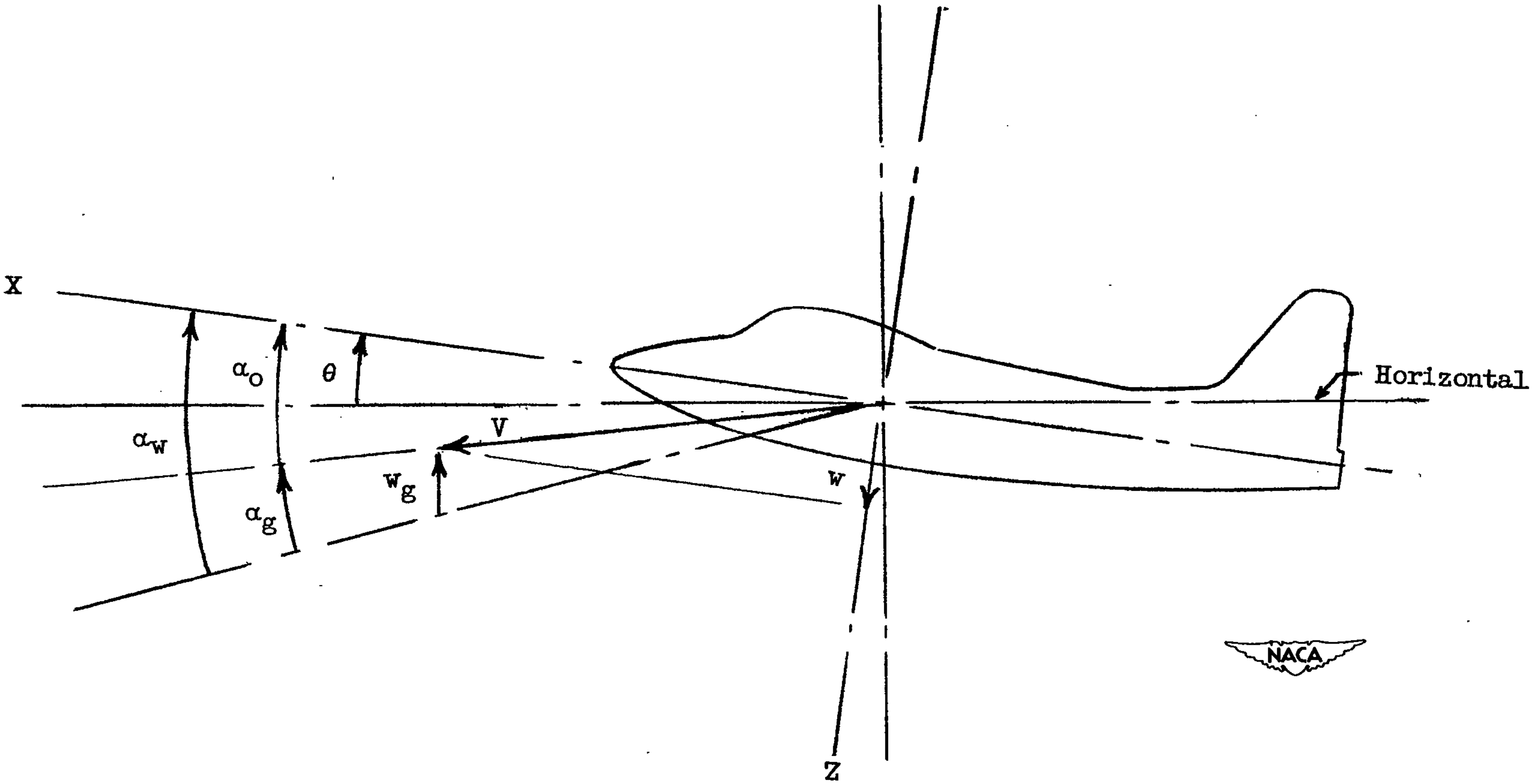


Figure 2.- Symbols and axes used in analysis. Positive directions of angles and velocities shown.

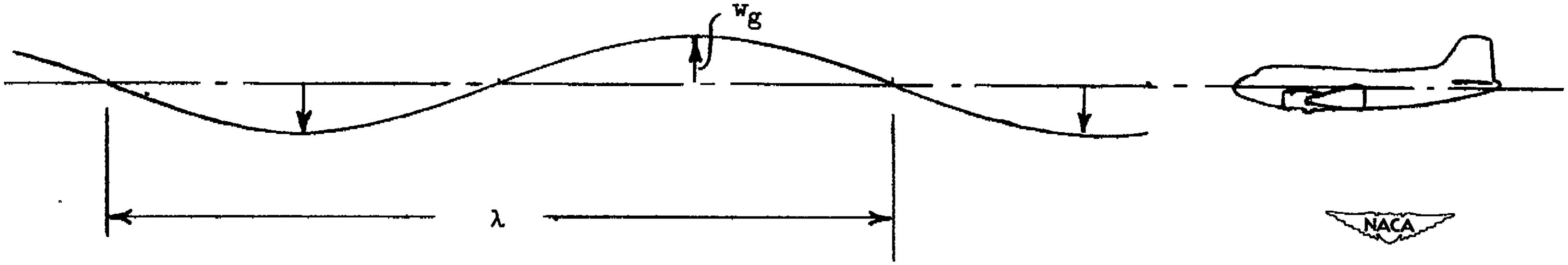


Figure 3.- Assumed form of sinusoidal gust disturbance.

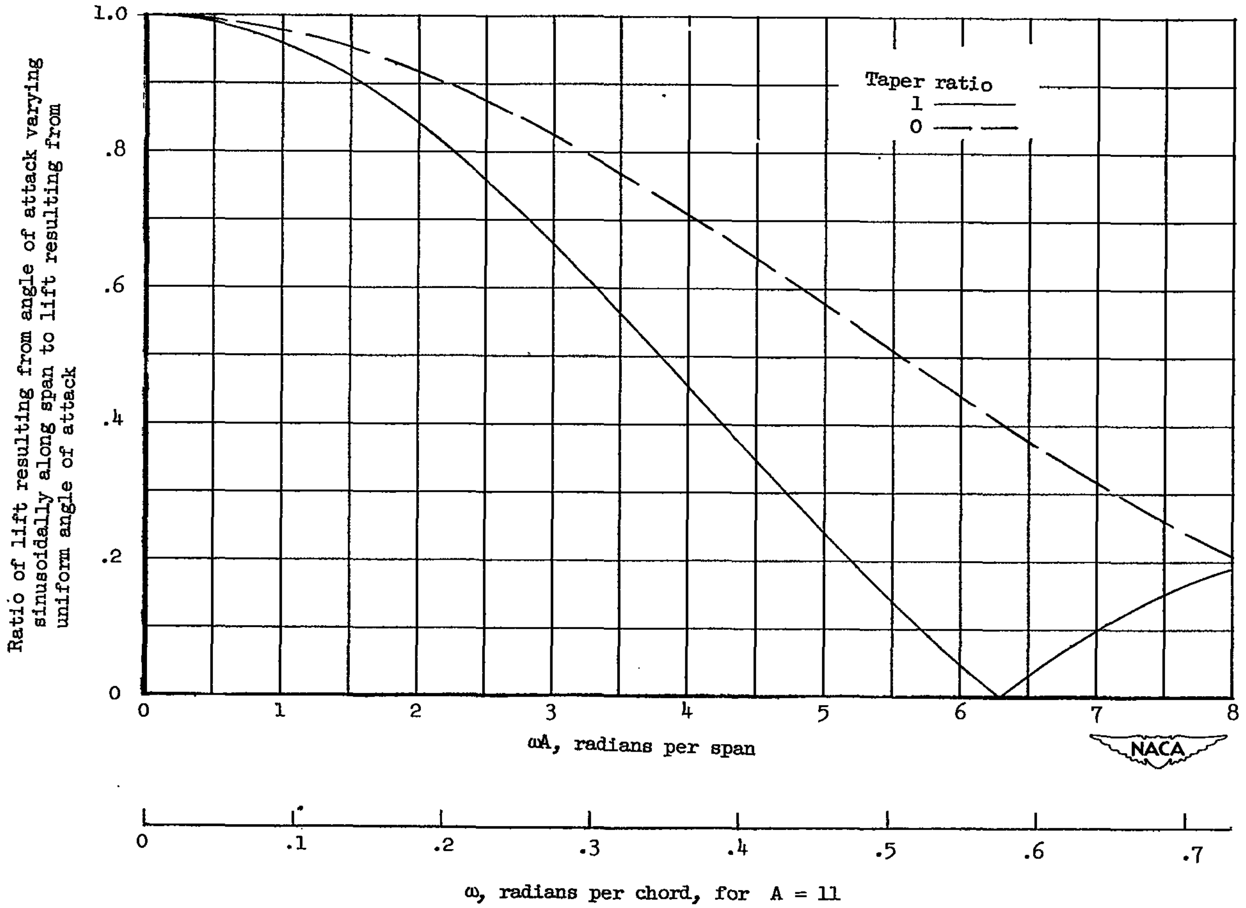


Figure 4.- Ratio of lift resulting from a gust angle of attack varying sinusoidally along the span to lift resulting from a uniform angle of attack, as a function of the quantity ωA. Curves based on strip-theory analysis are plotted for taper ratios 1 and 0. Spanwise orientation of gust is such as to produce the greatest lift for each gust wave length.

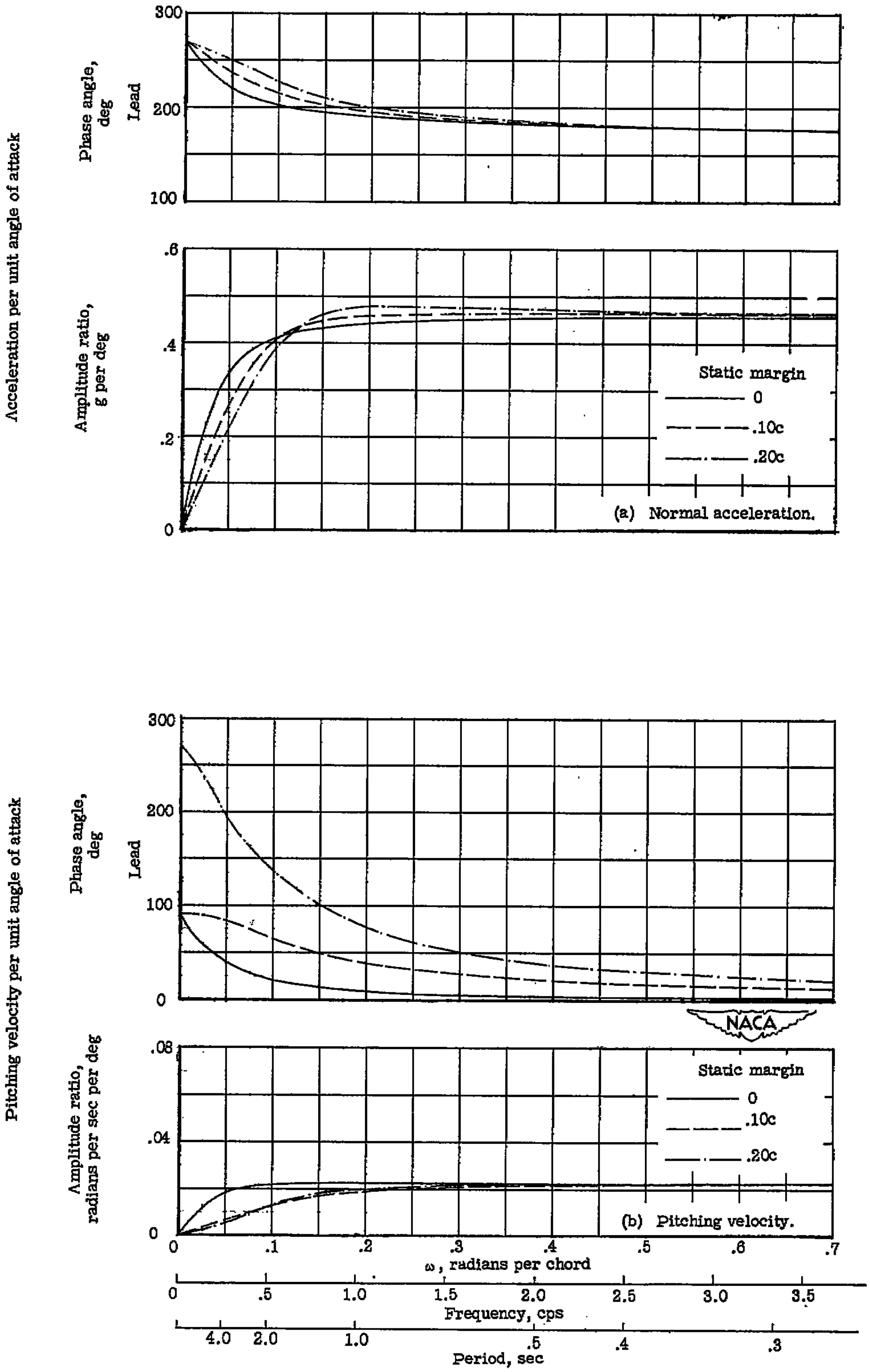


Figure 5.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for the basic airplane.

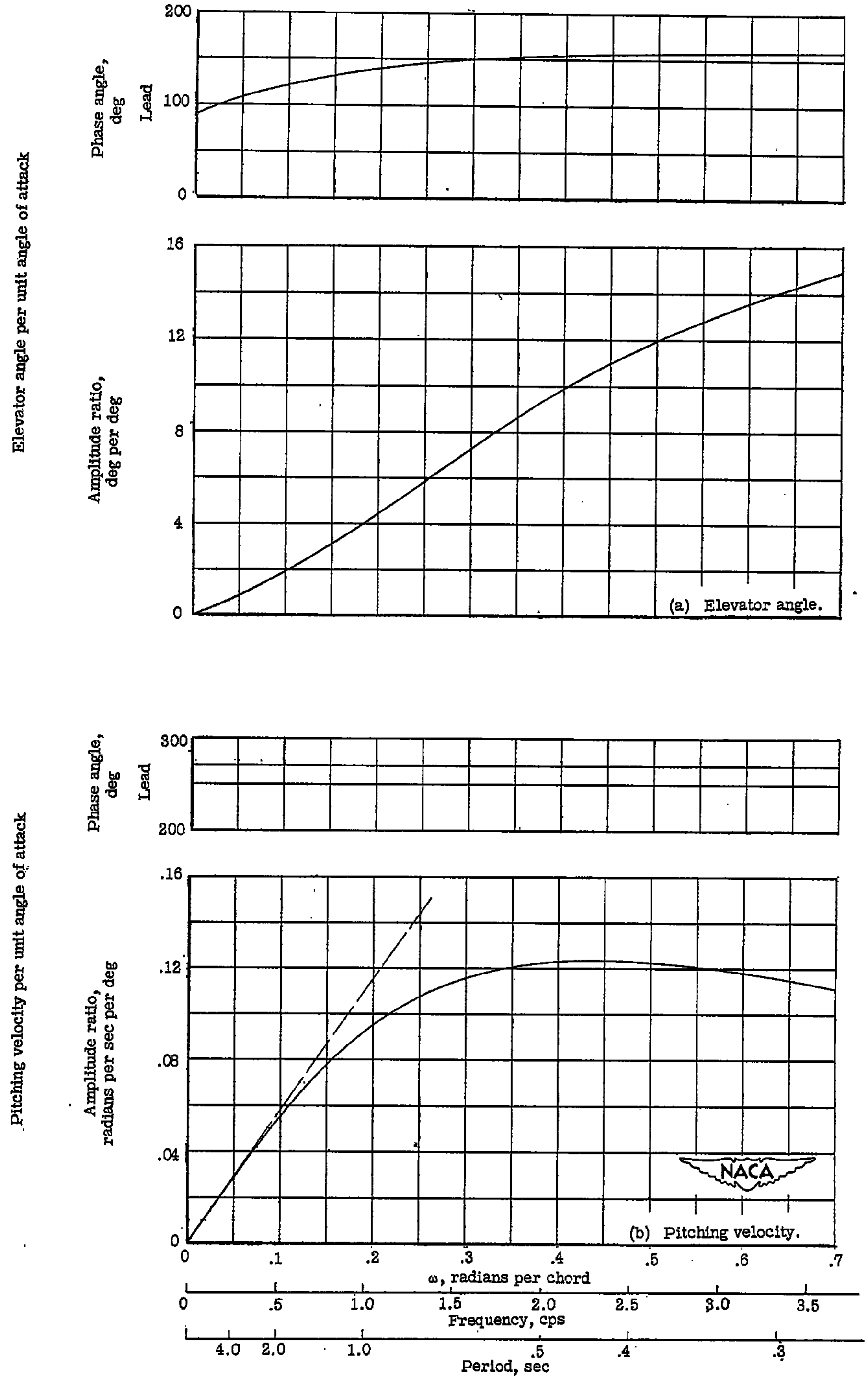


Figure 6.- Variation of elevator angle required to maintain zero normal acceleration at the center of gravity in flight through sinusoidal gusts and the resulting pitching velocity. Dashed line indicates pitching velocity required to maintain a constant angle of attack.

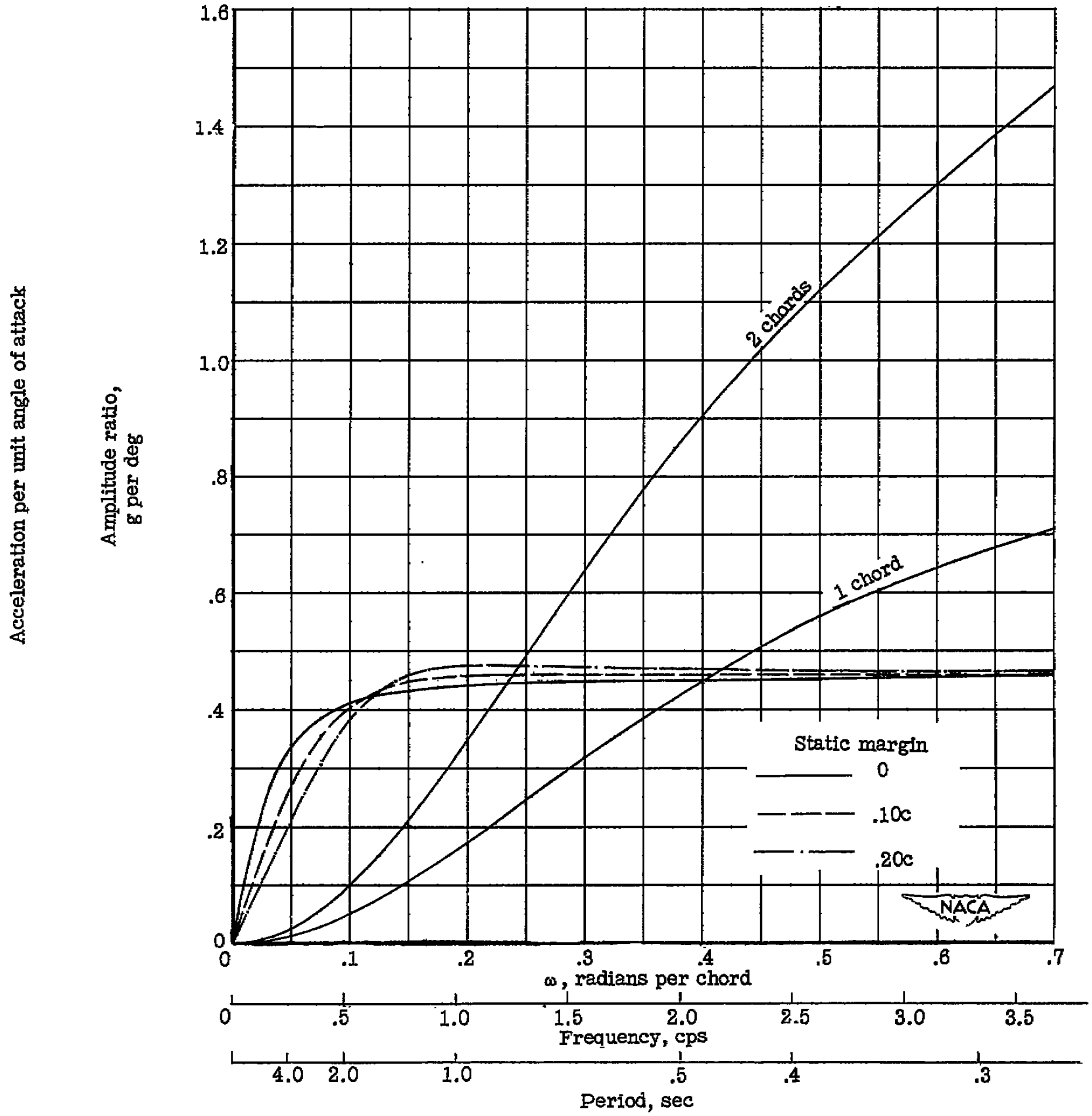
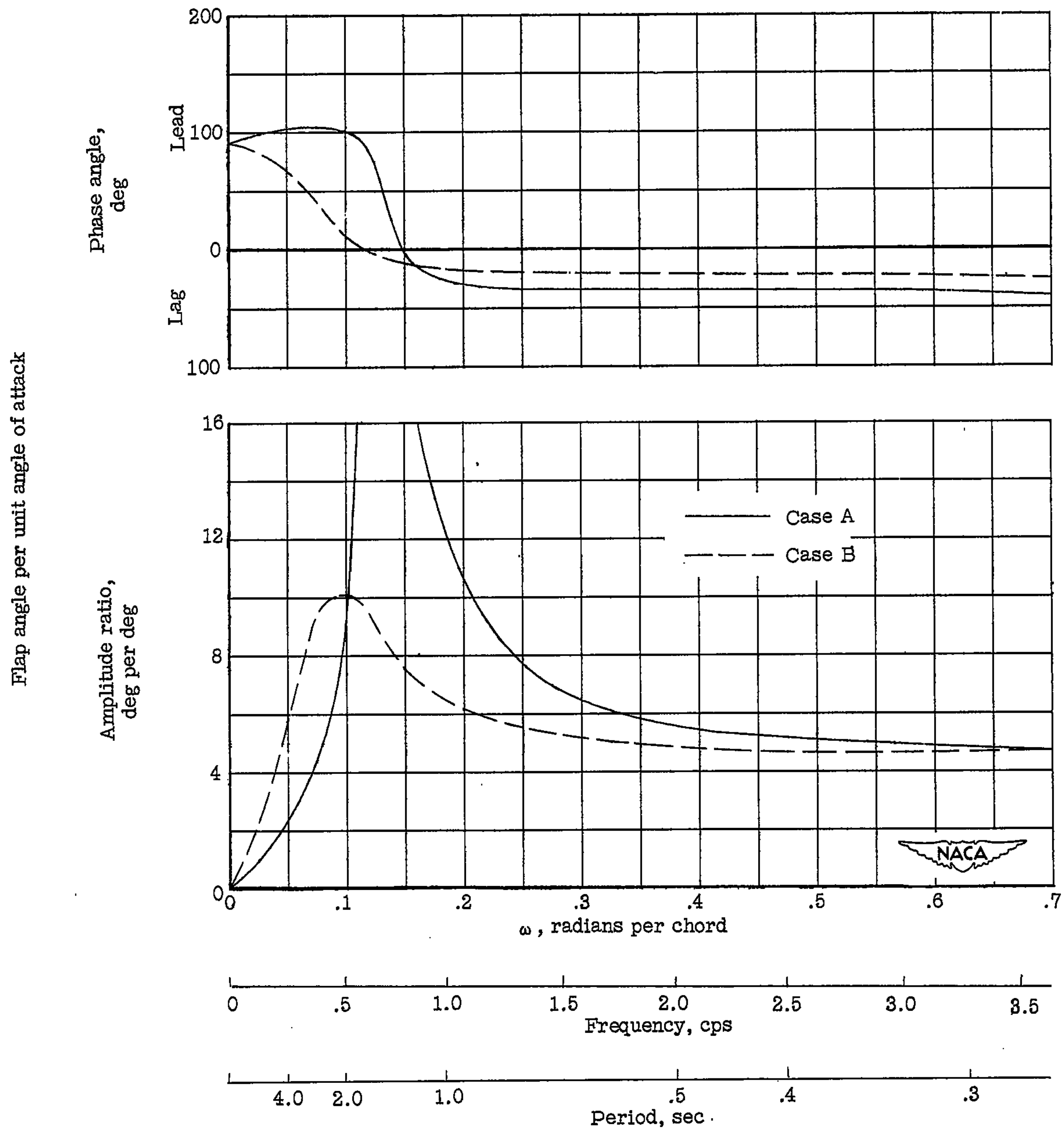
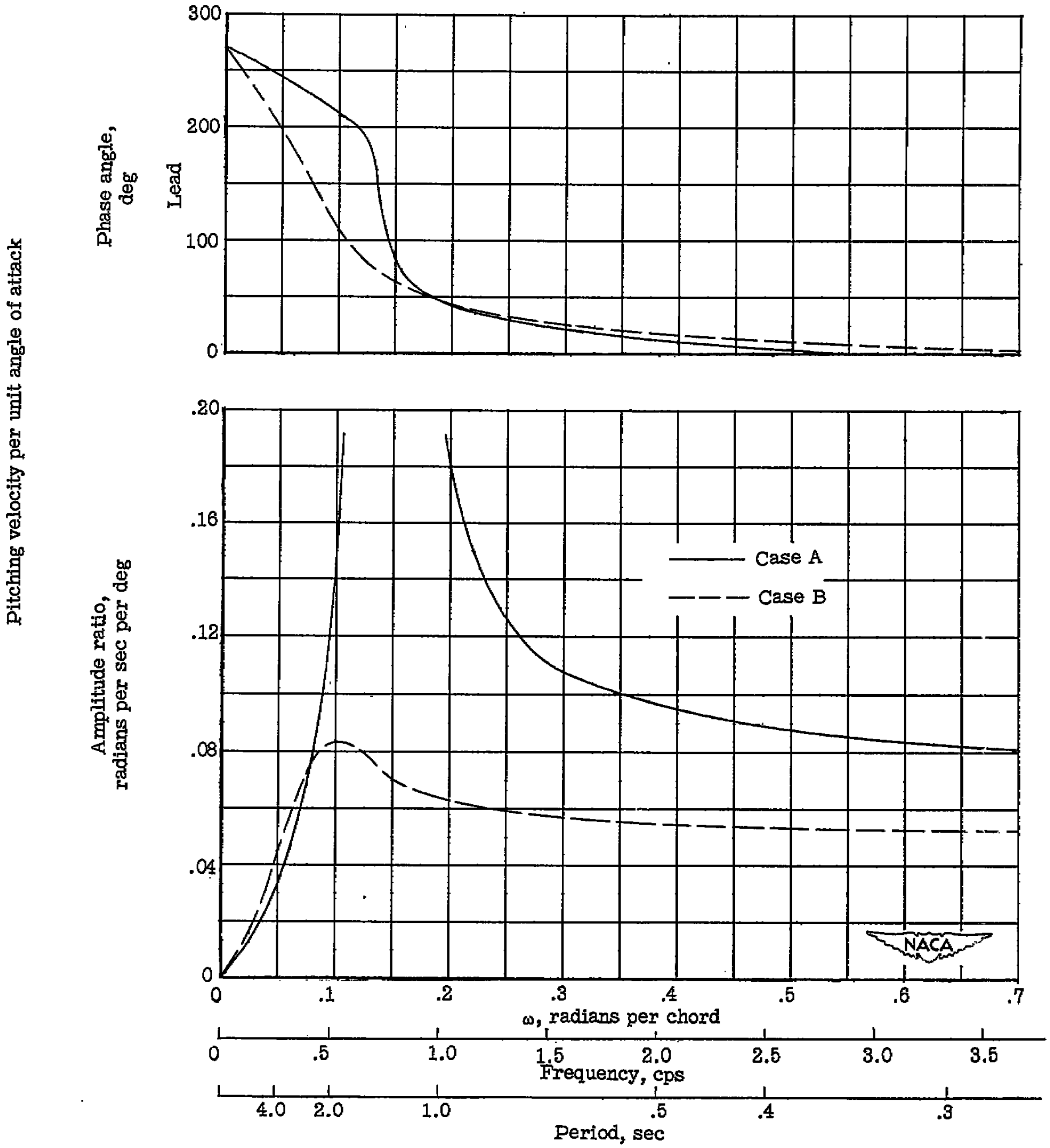


Figure 7.- Values of vertical acceleration at distances of 1 and 2 chords from the center of gravity caused by pitching oscillations when the elevator is operated to maintain zero acceleration of the center of gravity in flight through sinusoidal gusts of various frequencies. Values of acceleration of the center of gravity of the airplane without acceleration alleviator are shown for comparison.



(a) Flap angle.

Figure 8.- Variation of flap angle required to maintain zero normal acceleration of the center of gravity in flight through sinusoidal gusts and the resulting pitching velocity.



(b) Pitching velocity.

Figure 8.- Concluded.

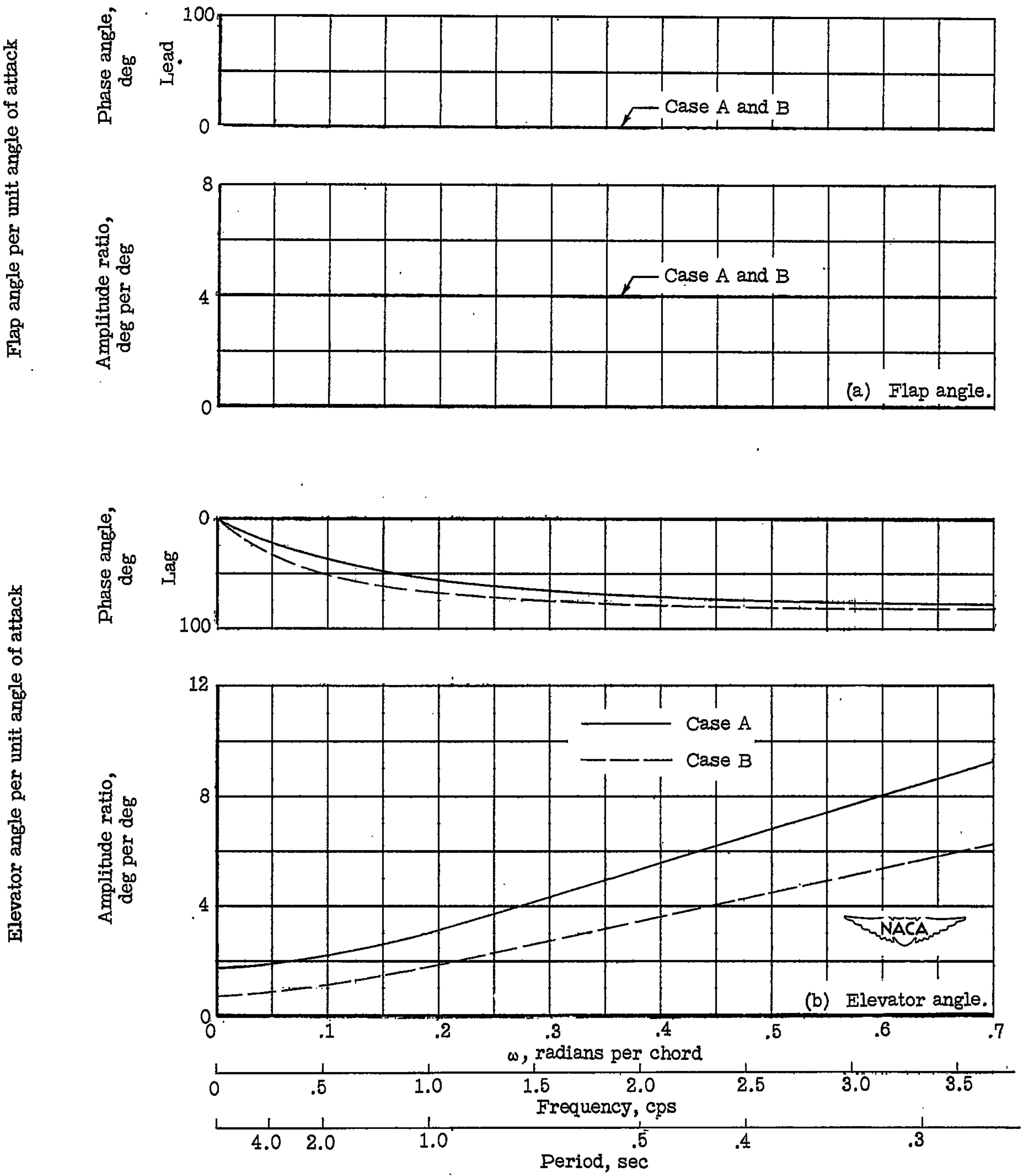


Figure 9.- Variation of the combined flap and elevator angles required to maintain zero normal acceleration of the center of gravity and zero pitching velocity in flight through sinusoidal gusts.

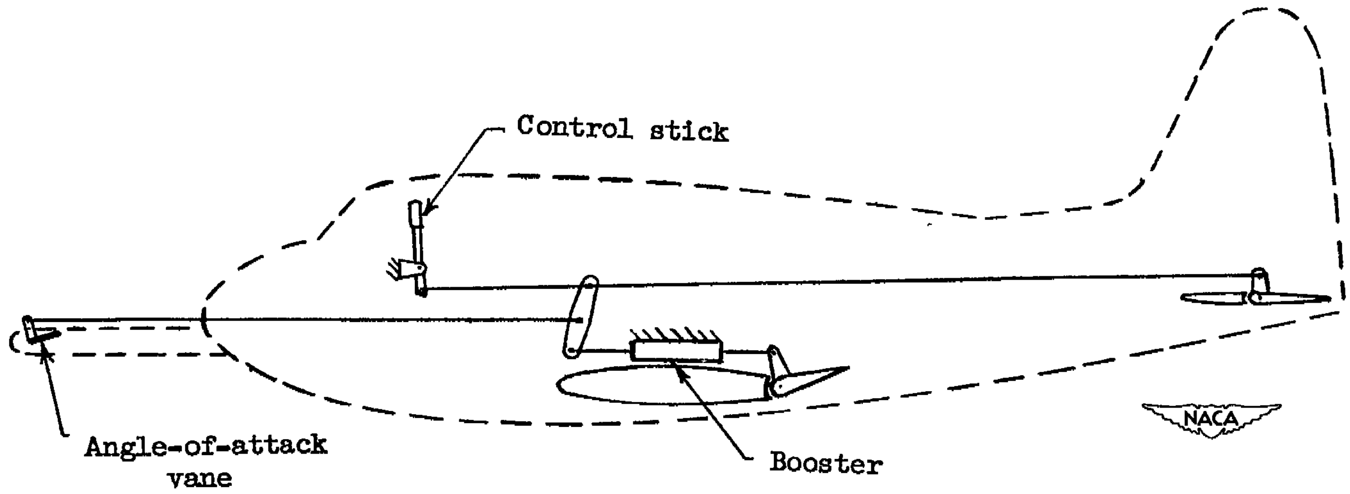
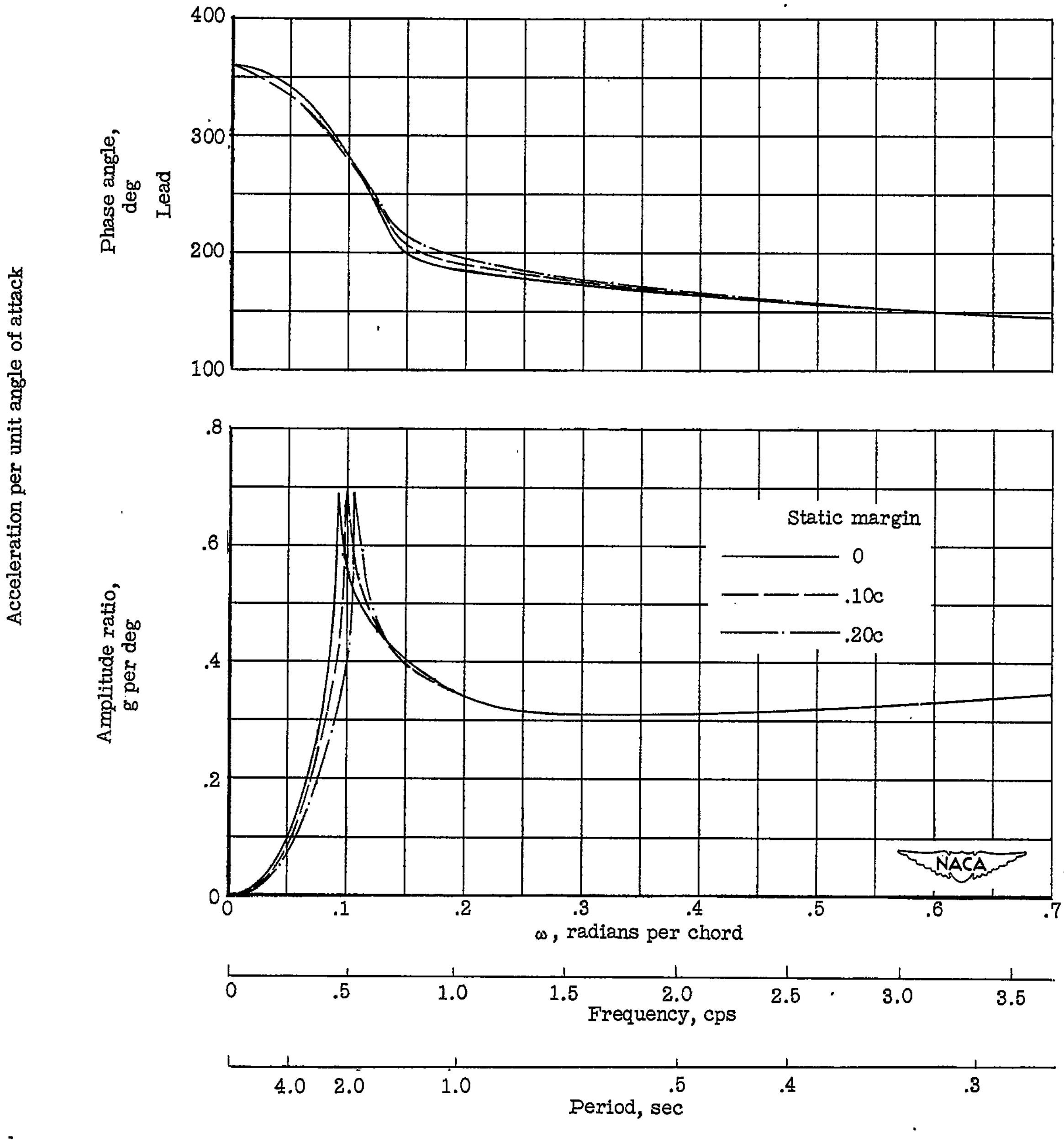
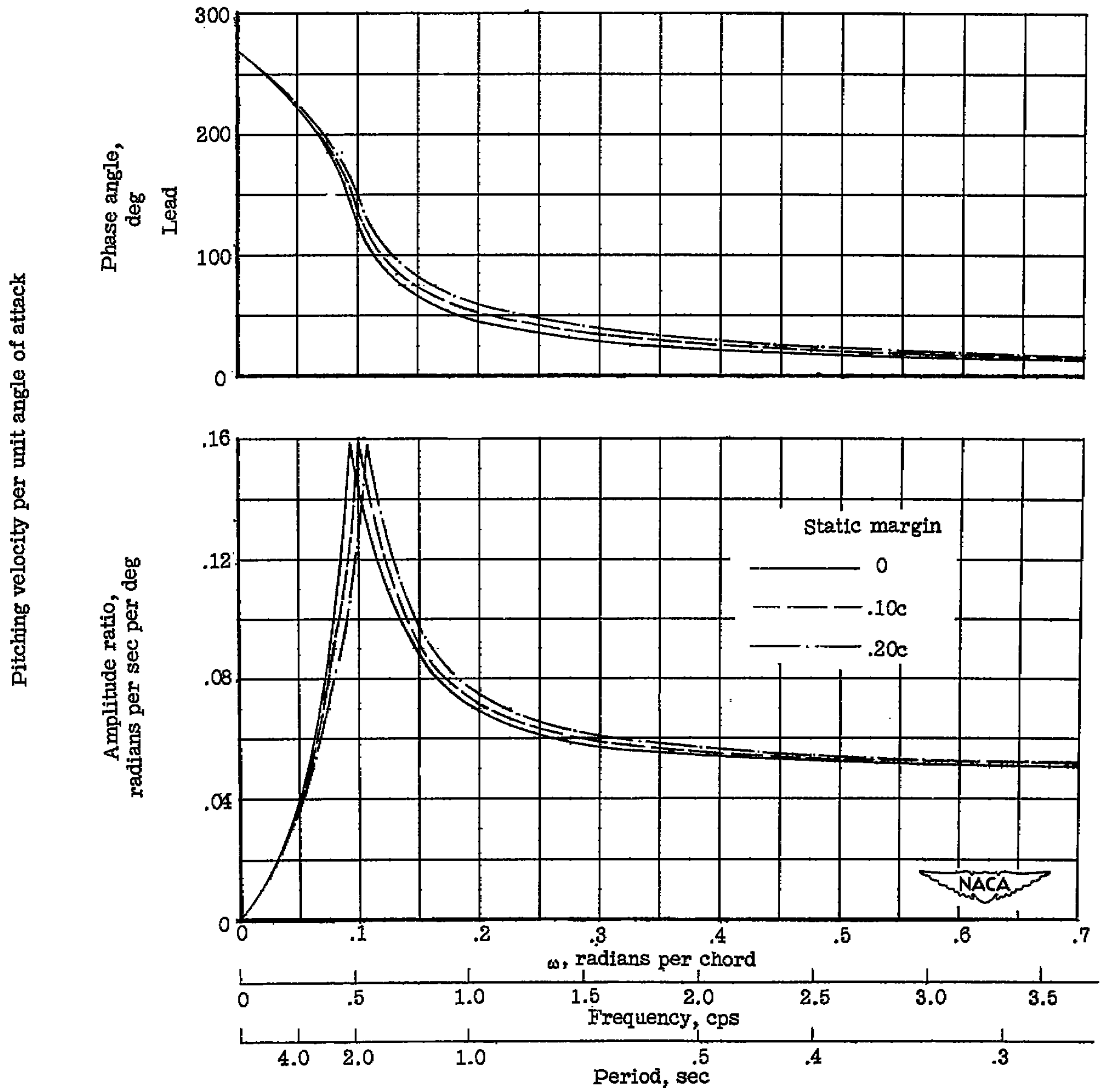


Figure 10.- Schematic diagram of mechanism for vane-controlled acceleration alleviator.



(a) Normal acceleration.

Figure 11.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the vane-controlled acceleration alleviator of case 1.



(b) Pitching velocity.

Figure 11.- Concluded.

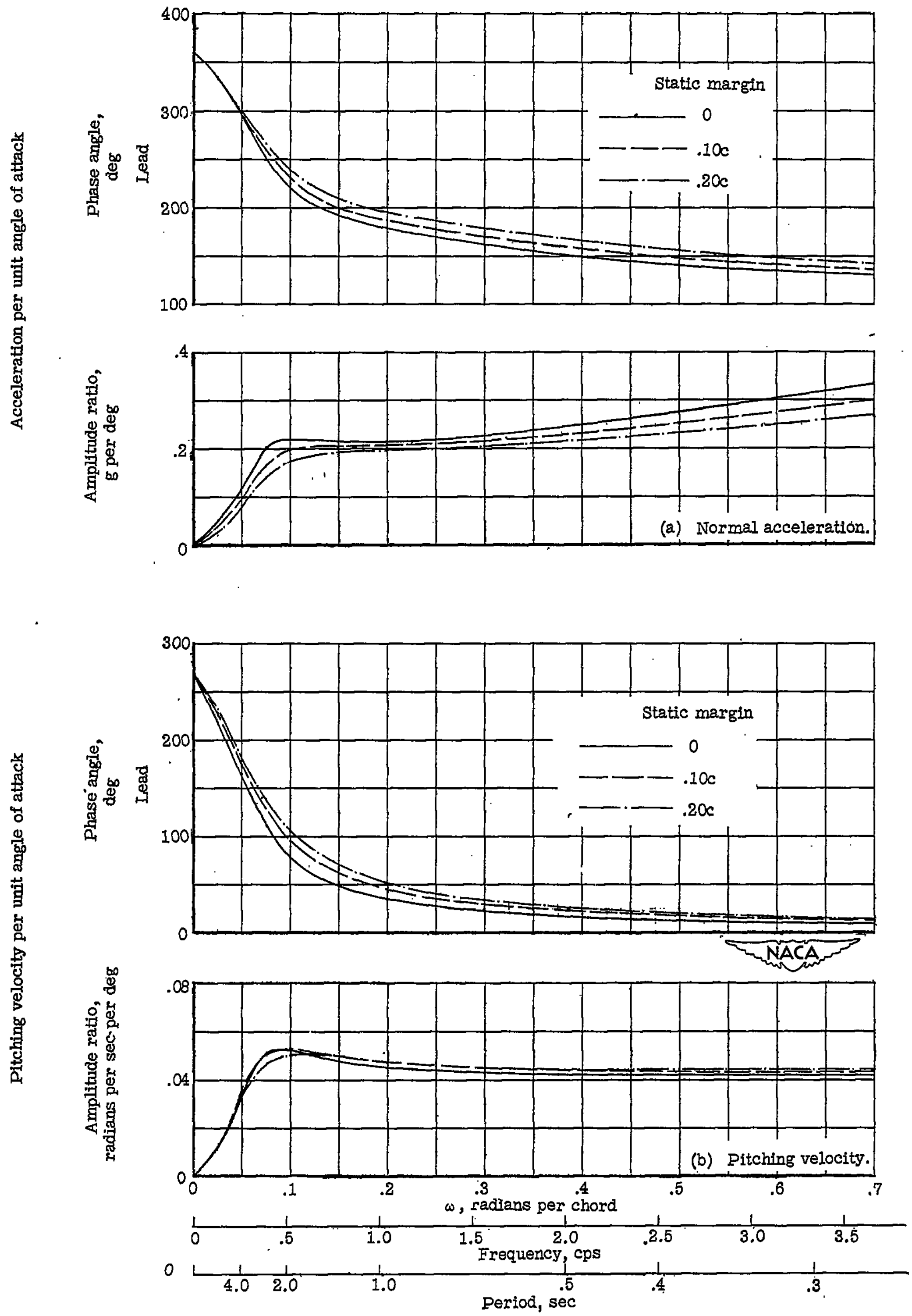


Figure 12.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the vane-controlled acceleration alleviator of case 2.

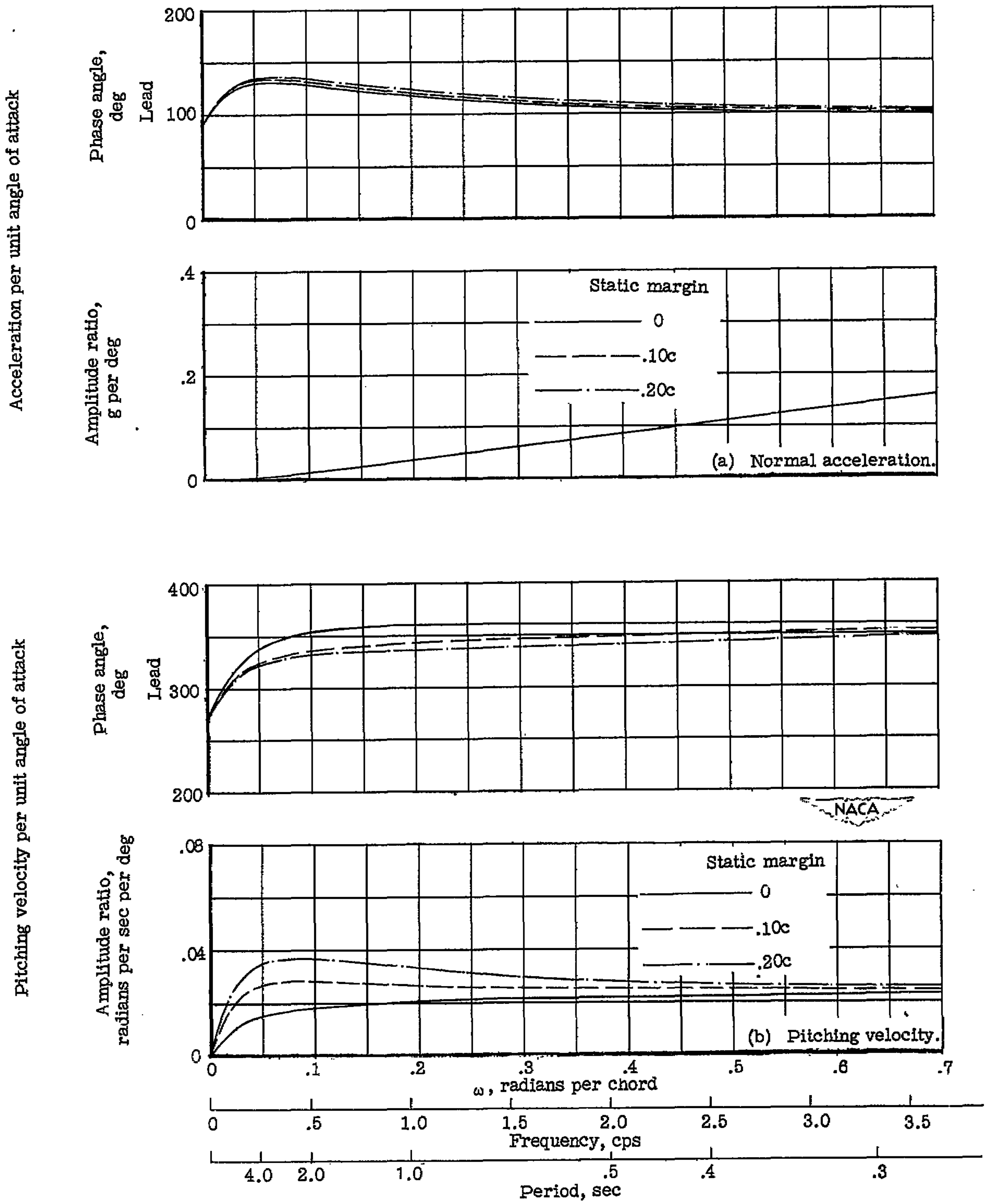


Figure 13.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the vane-controlled acceleration alleviator of case 3.

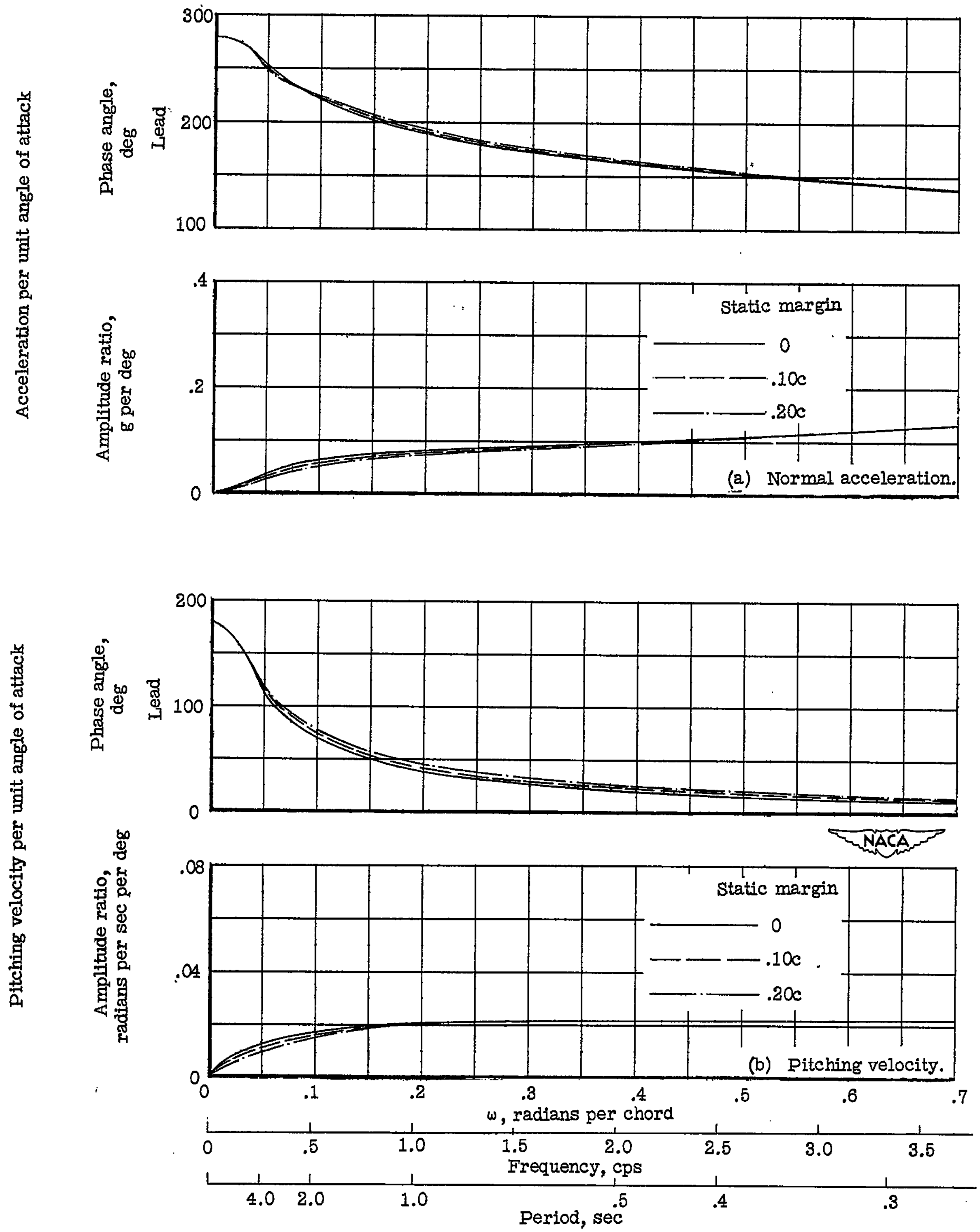


Figure 14.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the vane-controlled acceleration alleviator of case 5.

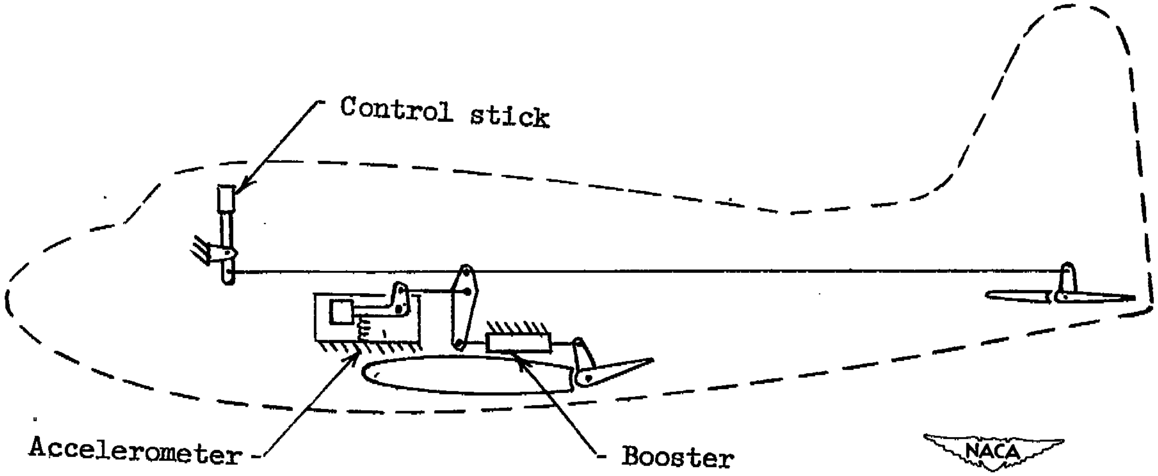


Figure 15.- Schematic diagram of mechanism for accelerometer-controlled acceleration alleviator.

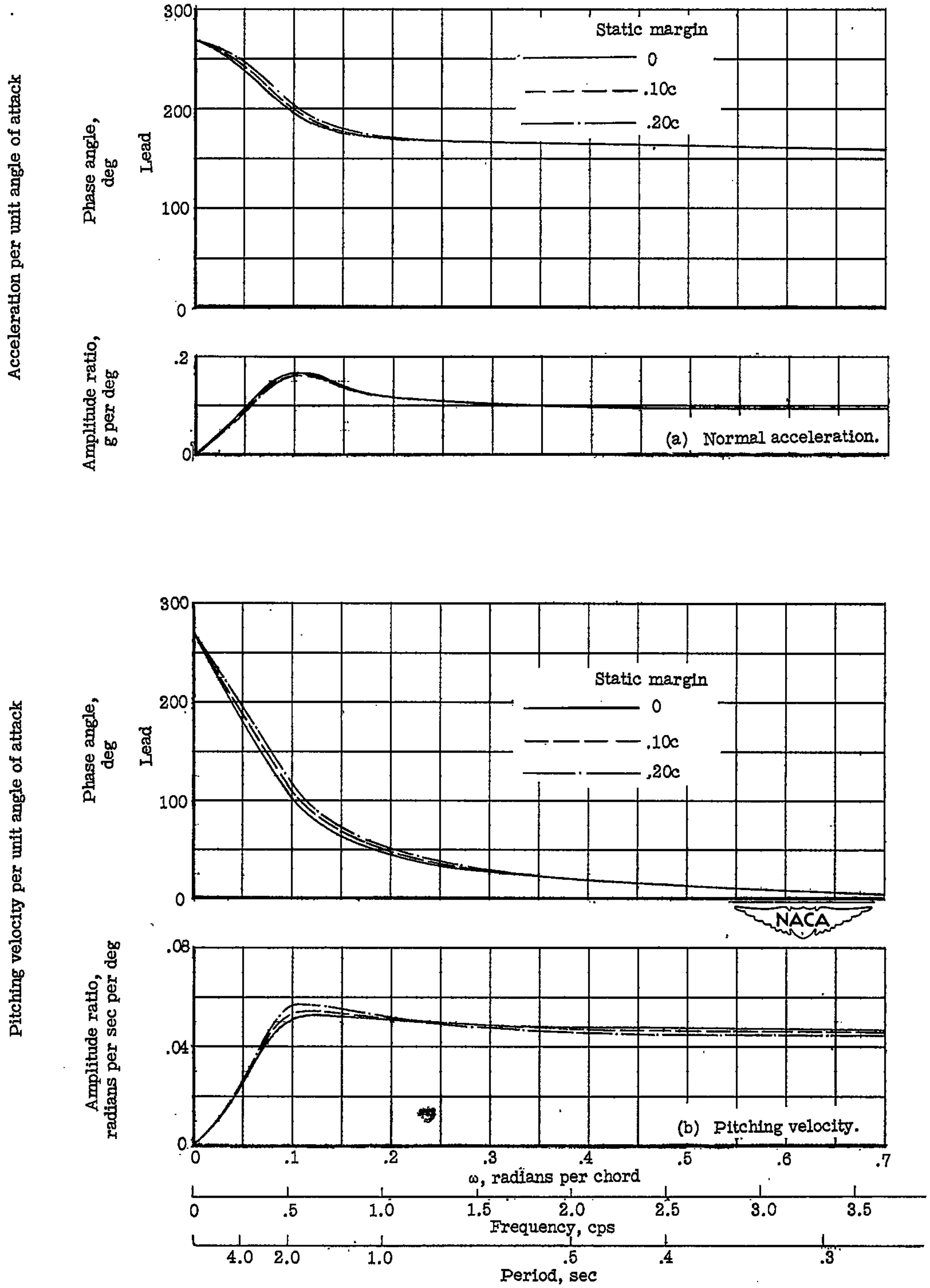


Figure 16.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the accelerometer-controlled acceleration alleviator of case 6.

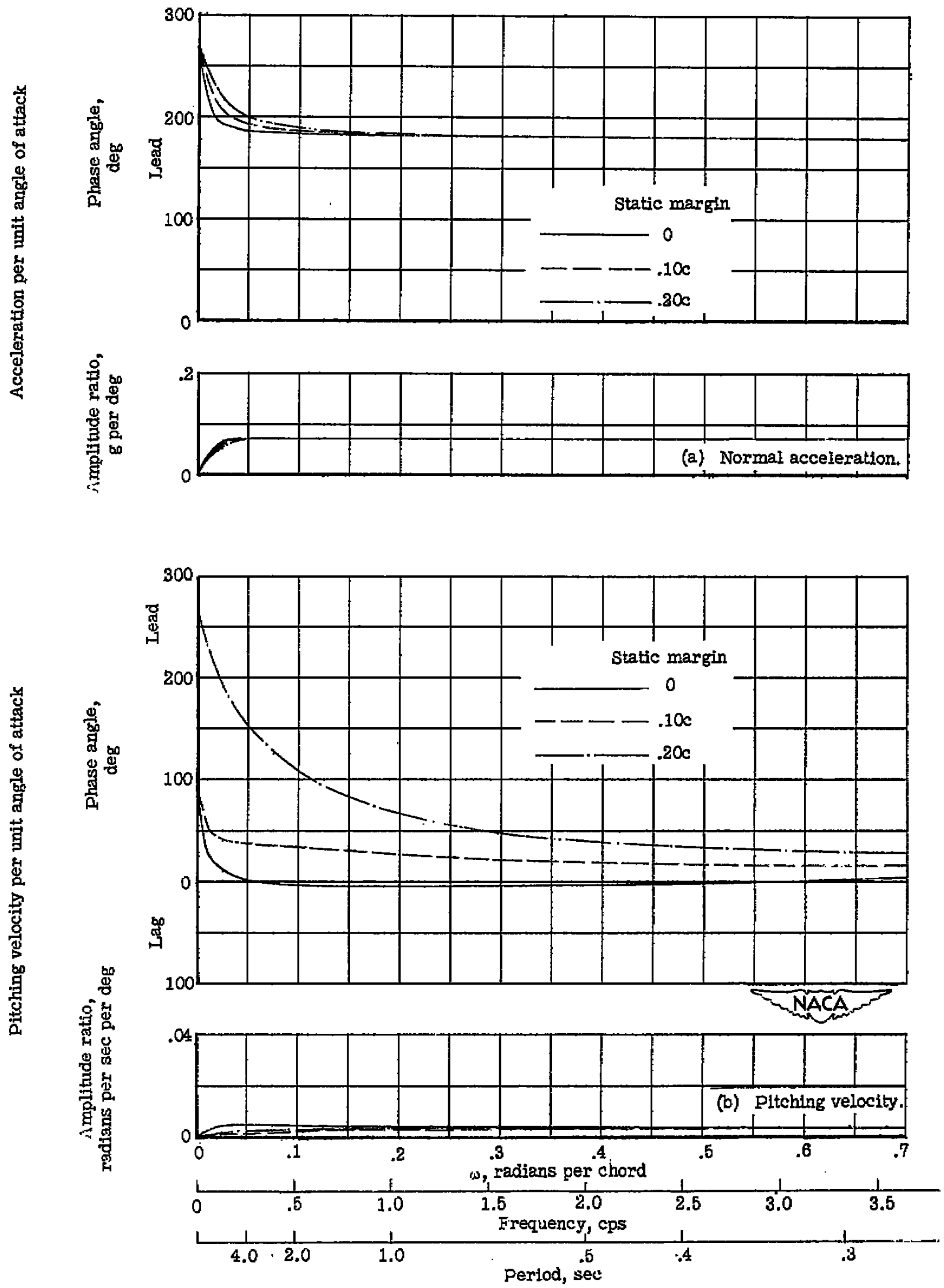


Figure 17.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the accelerometer-controlled acceleration alleviator of case 7.

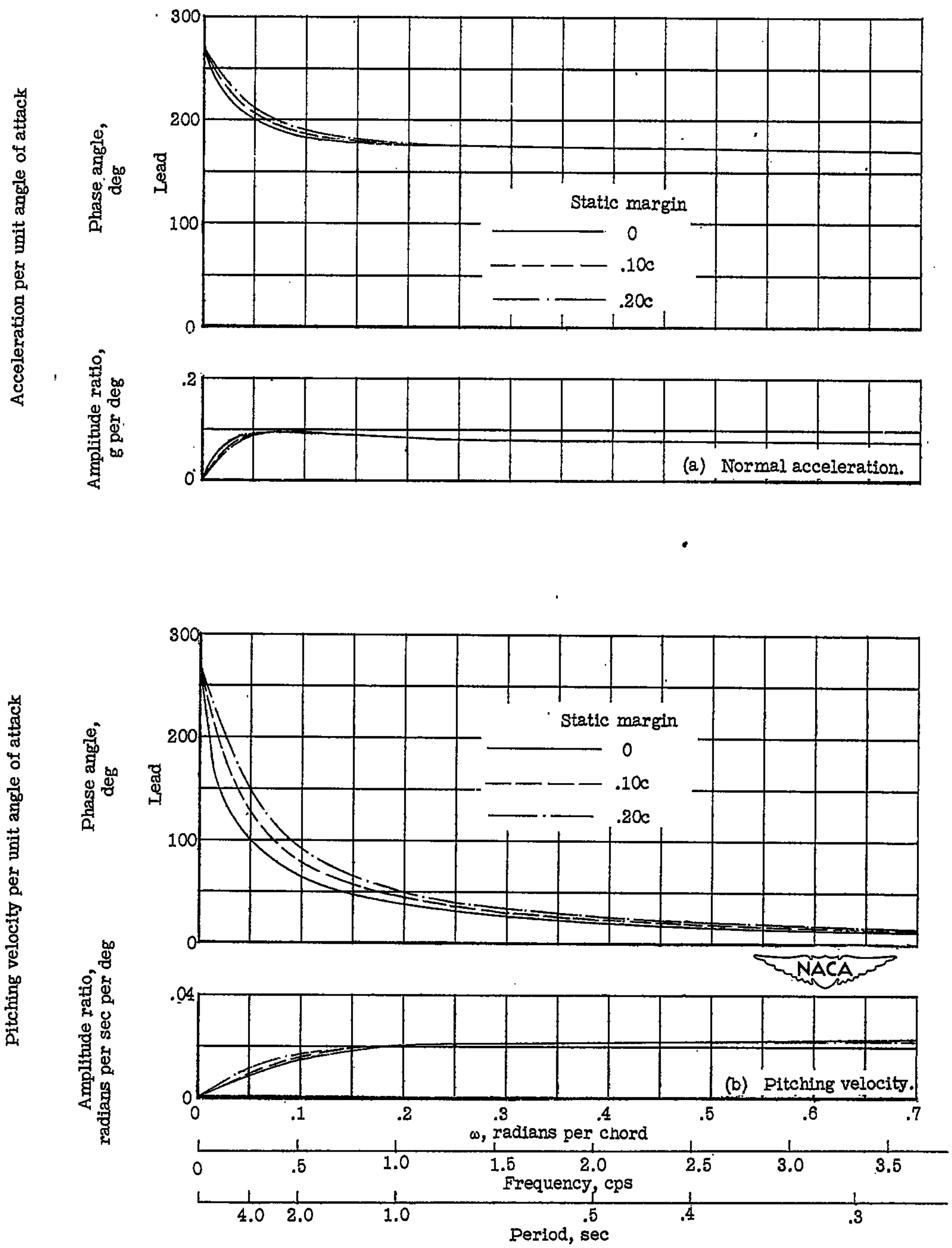


Figure 18.- Variations with gust frequency of the normal acceleration and pitching velocity resulting from sinusoidal gusts for an airplane with the accelerometer-controlled acceleration alleviator of case 8.

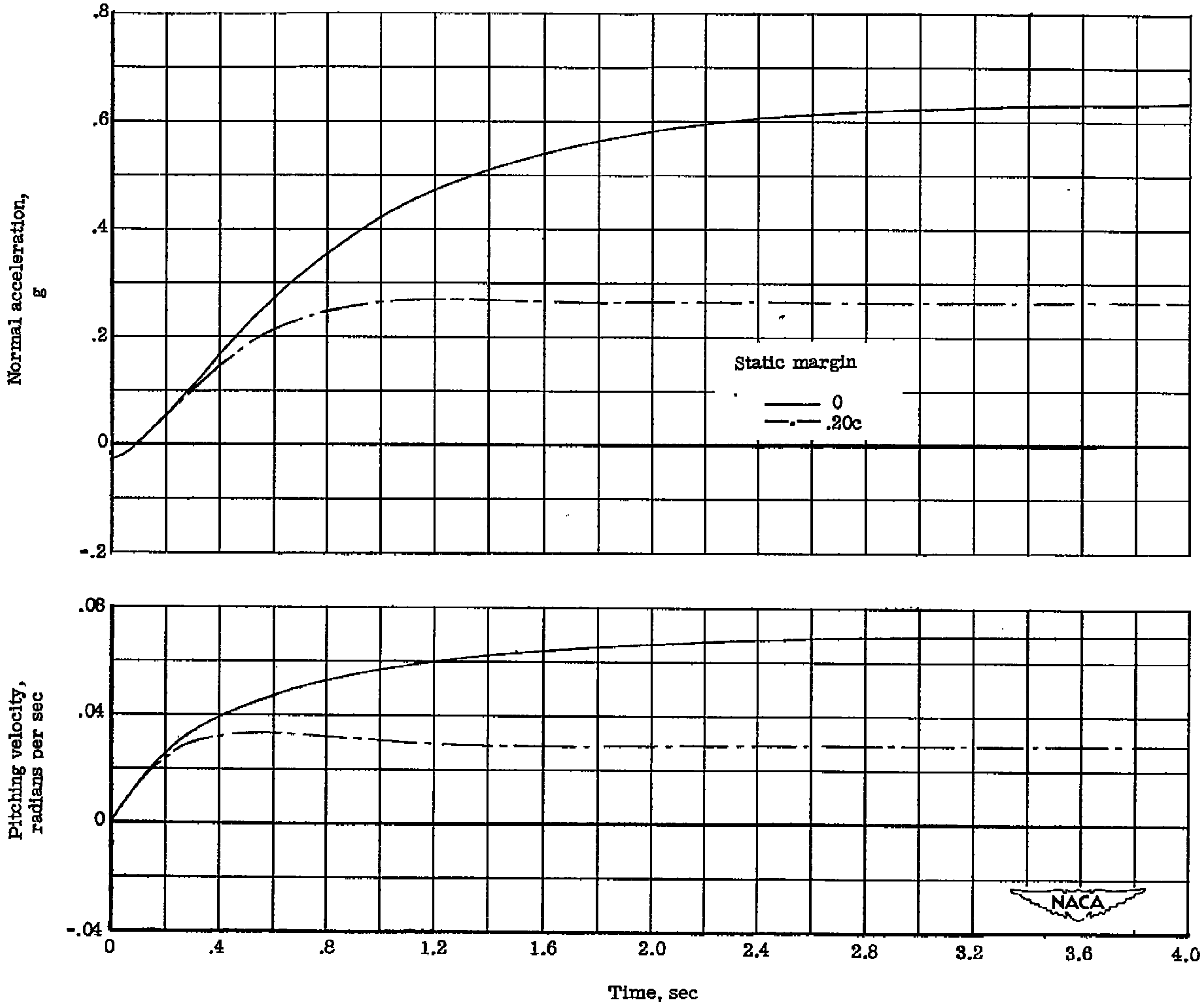


Figure 19.- Response of basic airplane to a step motion of the elevator of 1°.

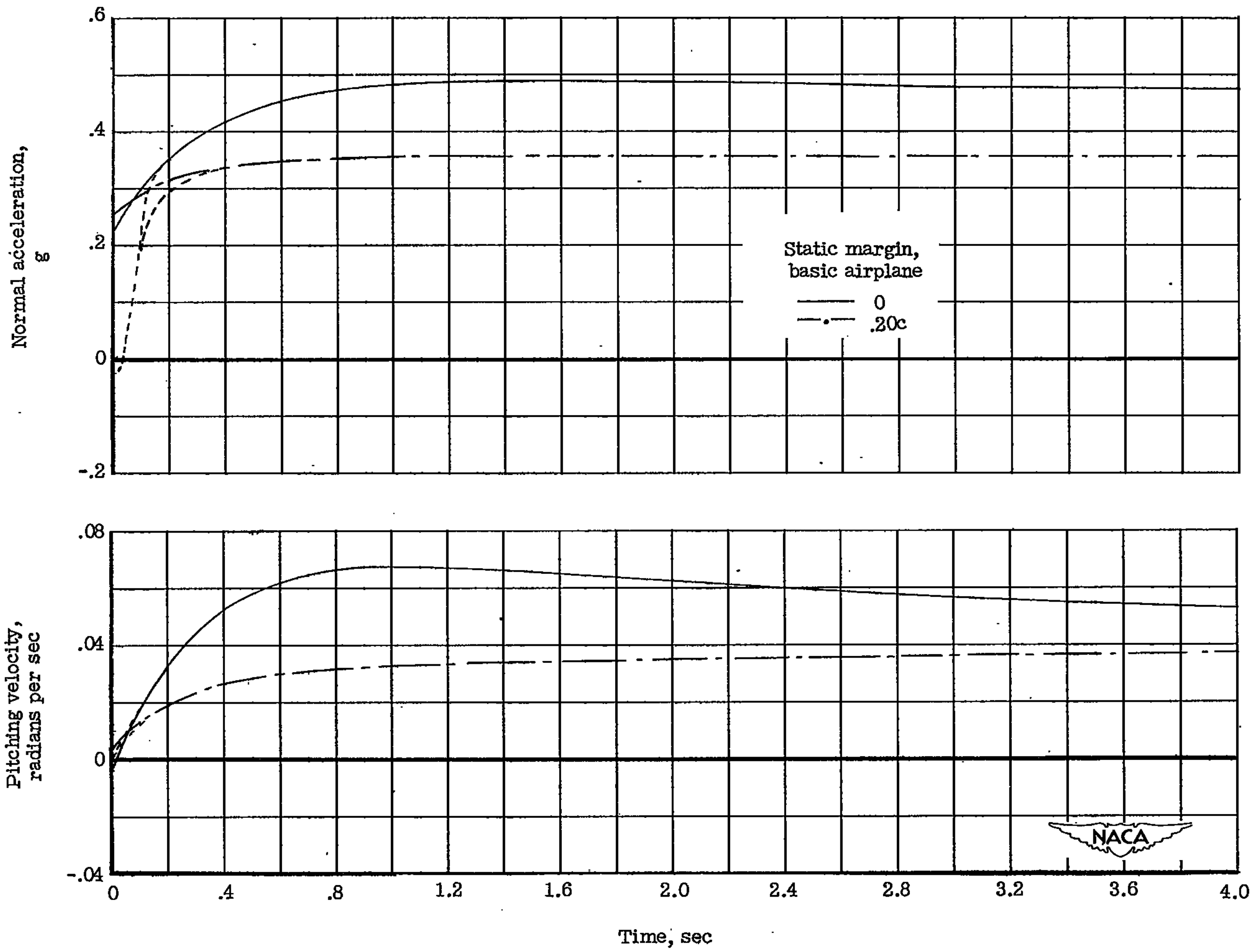


Figure 20.- Response of airplane with vane-controlled acceleration alleviator of case 5 to a step motion of the elevator of 1°. Theoretical solution shows impulsive negative values of acceleration at $t = 0$. Dotted curves indicate approximate expected variation of acceleration near $t = 0$.

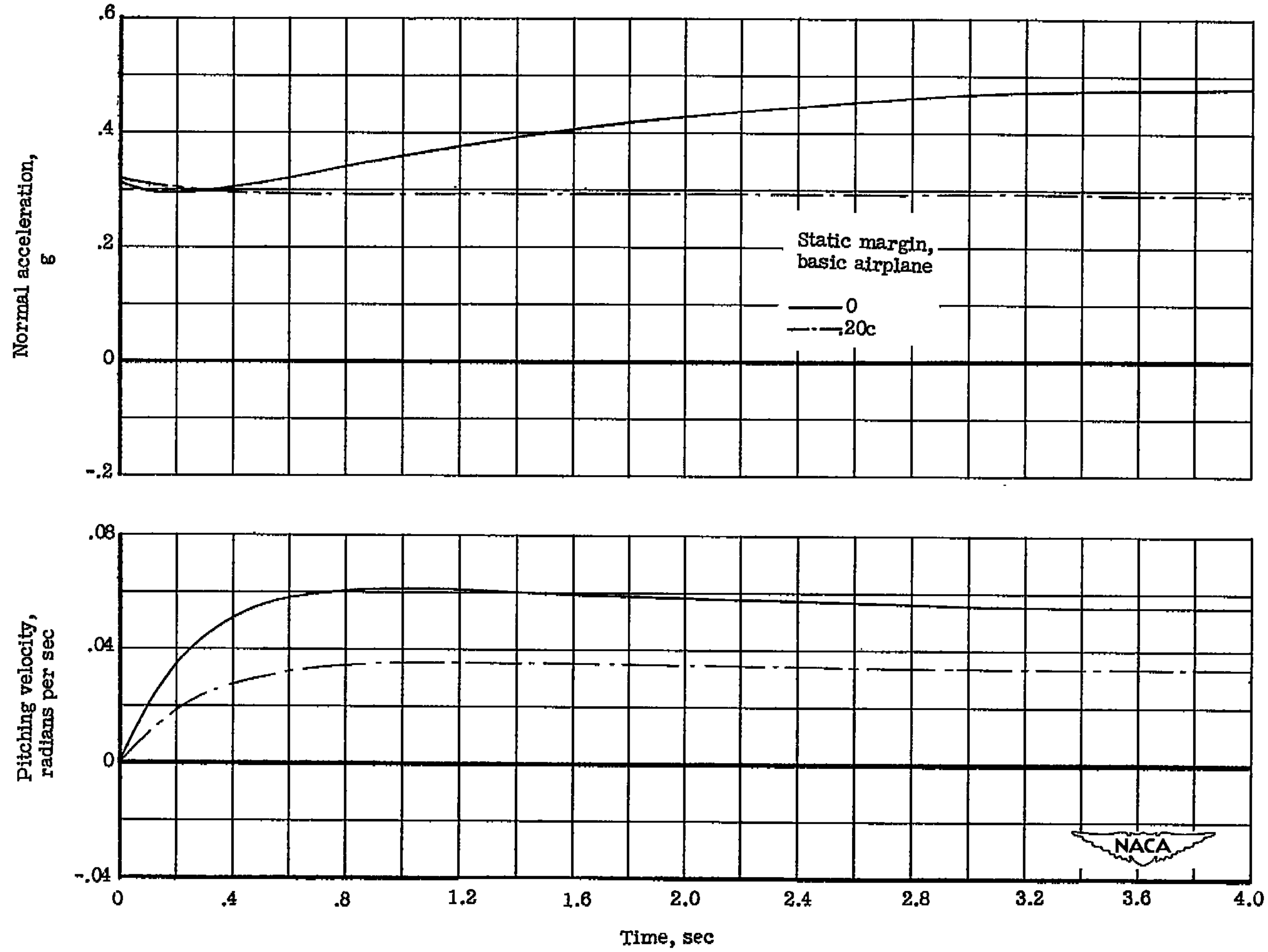


Figure 21.- Response of airplane with accelerometer-controlled acceleration alleviator of case 8 to a step motion of the elevator of 1°.

NACA-Langley - 7-23-51 - 1000